

Calculation Policy

Introduction

This calculation policy is designed to ensure consistency and progression in the teaching of addition, subtraction, multiplication and division across the school. It is aligned with the 2014 National Curriculum.

Children will use mental calculation approaches as their first port of call when it is efficient and appropriate to do so. When necessary, an efficient written method needs to be used accurately, confidently and with clear understanding.

Within each section there are examples of concrete (the practical items that pupils can hold and manipulate to help them explore abstract mathematical concepts and the relationships between them), pictorial (models and representations) and abstract (the symbolic stage).

Concrete

Concrete is the “doing” stage, using concrete objects to model problems. Instead of the traditional method of maths teaching, where a teacher demonstrates how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical objects themselves. Every new abstract concept is learned first with a “concrete” or physical experience.

For example, if a problem is about adding up four baskets of fruit, the children might first handle actual fruit before progressing to handling counters or cubes which are used to represent the fruit.

Pictorial

Pictorial is the “seeing” stage, using representations of the objects to model problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem.

Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult, such as fractions, as it helps them visualise the problem and make it more accessible.

Abstract

Abstract is the “symbolic” stage, where children are able to use abstract symbols to model problems (Hauser).

Only once a child has demonstrated that they have a solid understanding of the “concrete” and “pictorial” representations of the problem, can the teacher introduce the more “abstract” concept, such as mathematical symbols. Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols, for example $+$, $-$, \times , \div to indicate addition, multiplication, or division.

Although presented as three distinct stages, it would be expected for teaching to go back and forth between each representation to reinforce concepts.

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ADDITION AND SUBTRACTION

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ADDITION & SUBTRACTION

Year 1

Number – addition and subtraction

Statutory requirements

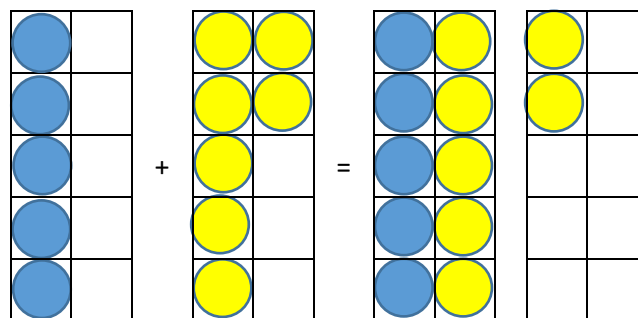
Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Year 1 Addition

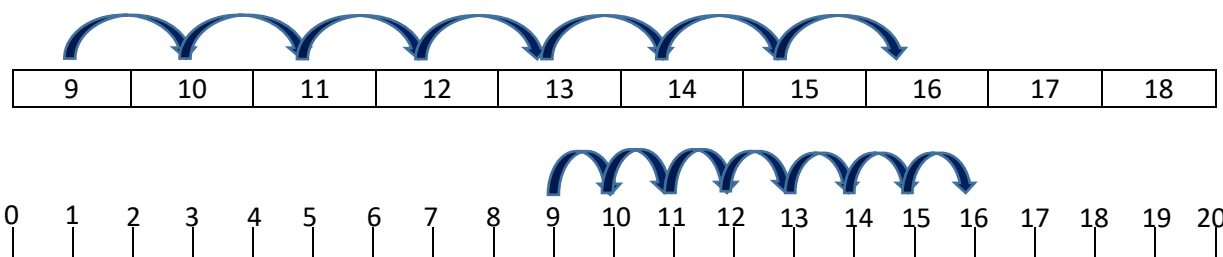
Concrete

Ten Frames are used to develop the ‘sense of number’ and identify the number bonds within 10 and up to 20, a great deal of use is made of 10 frames, looking at patterns within the numbers. This leads into discussion of how to look at an addition such as $5 + 7$.



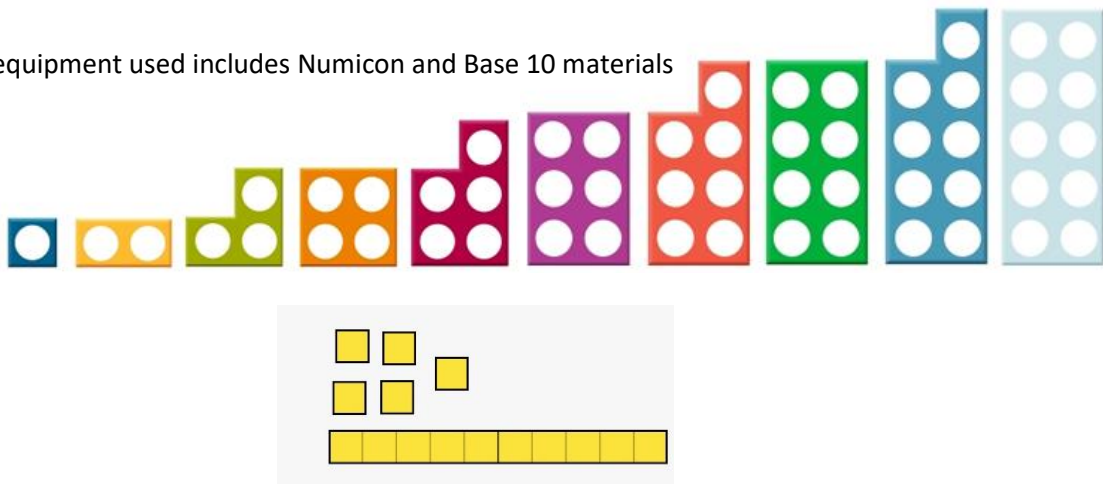
Pupils count on with number tracks/number lines

$$9 + 7 = \square$$



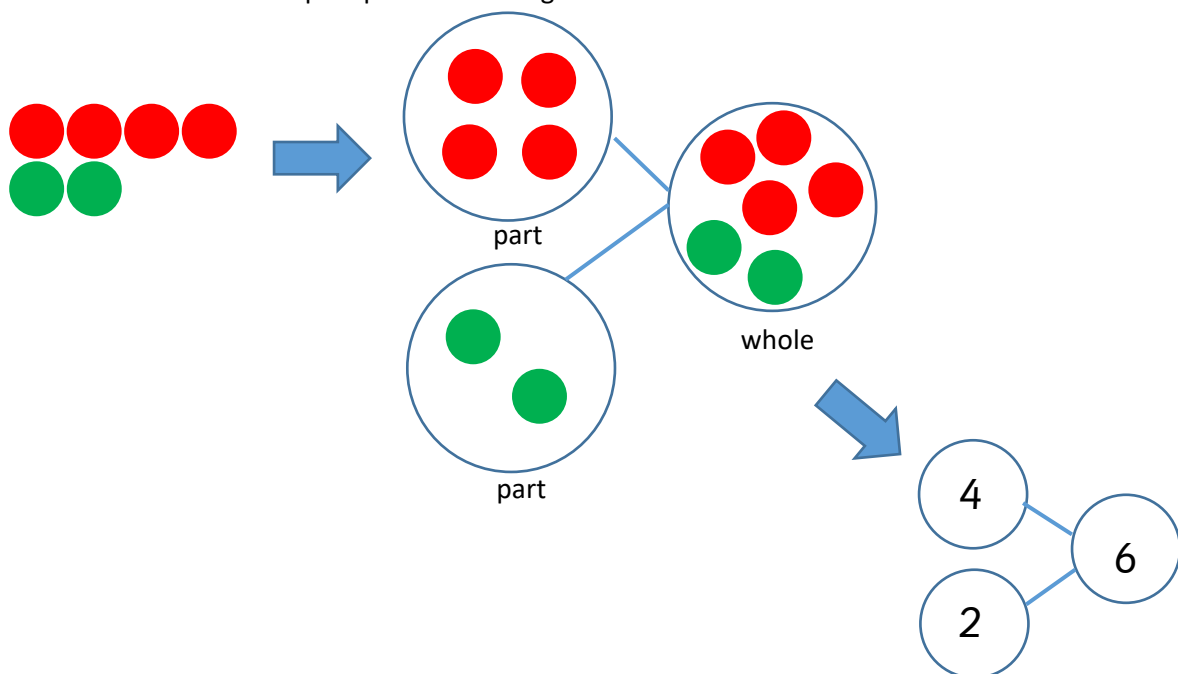
Lots of activities take place with physical resources (counters, cubes, coins, toy characters etc.)

Other equipment used includes Numicon and Base 10 materials



Pictorial

Extensive use is made of part-part-whole Diagrams:



Abstract

The introduction of addition stories.

Introduction of $\square = 9 + 7$ to demonstrate the meaning of the equals sign in balanced equations (reinforced with practical examples using scales) and commutative number sentences showing that $4+2$ is the same as $2+4$.

Year 1 Subtraction

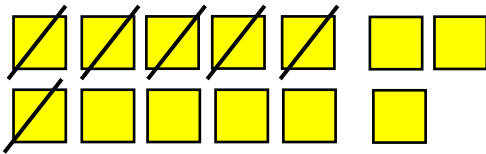
Concrete

The same range of resources are used as with addition.

Pictorial

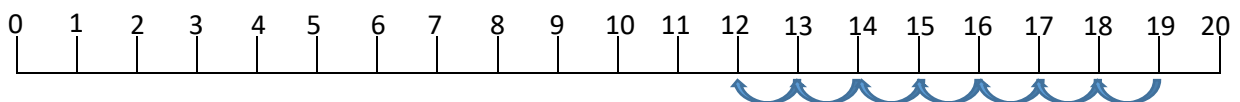
Subtraction by crossing out

$$13 - 6 = \square$$



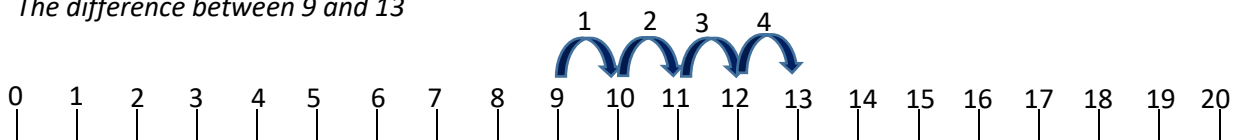
Subtraction by counting back

$$19 - 7 = \square$$



Counting on to find the difference between two numbers:

The difference between 9 and 13

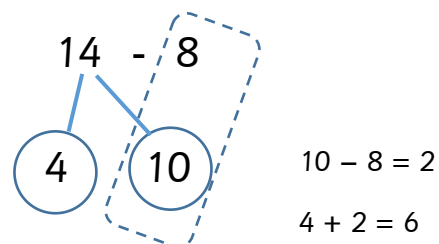
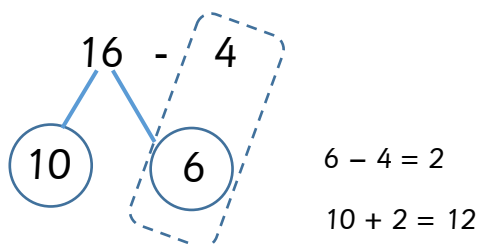


Abstract

The introduction of subtraction stories.

Start to explore missing number problem involving – and = notation.

Manipulate numbers with part-part-whole to make calculations easier:



Year 2

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
 - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
 - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

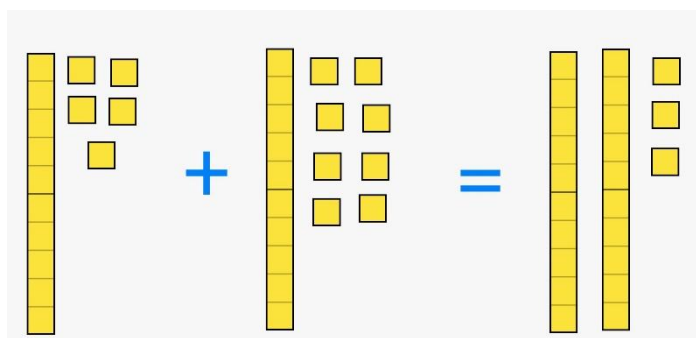
Year 2 Addition

Concrete

Continue to use resources introduced in Year 1.

In addition to ten frames, card versions are also used.

A greater use is made of Base 10 materials to reinforce the concepts behind the regrouping and renaming when crossing the boundary between ones and tens.

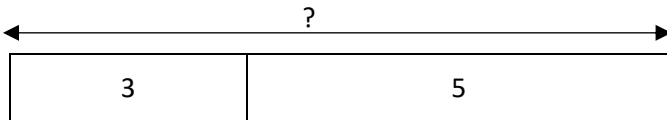


Pictorial

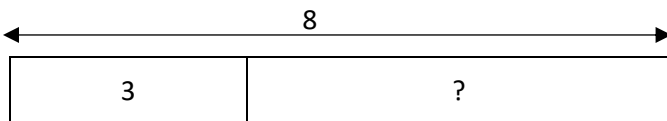
The methods engaged with the Base 10 equipment is 'illustrated' in pupil diagrams.

Year 2 sees an introduction to bar modelling:

$$3 + 5 = \square$$

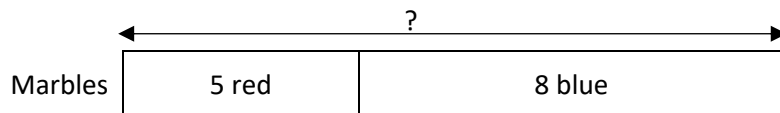


$$3 + \square = 8$$

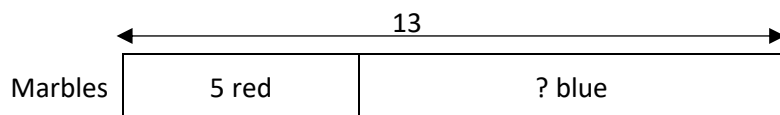


Extending to context based problems such as:

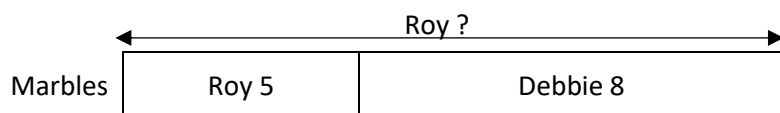
Roy has 5 red marbles and 8 blue marbles. How many marbles does he have altogether? (*Whole unknown*)



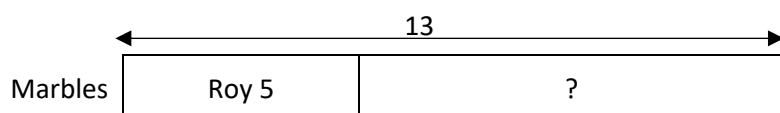
Roy has 13 marbles. Five are red and the rest are blue. How many blue marbles does Roy have? (*Part unknown*)



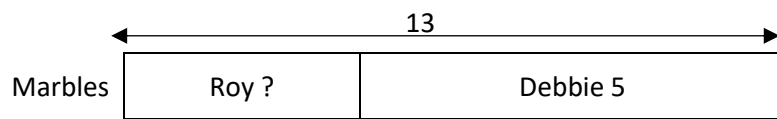
Roy had 5 marbles. Debbie gave him 8 more marbles. How many marbles does Roy have now? (*Result unknown*)



Roy has 5 marbles. How many more marbles does he need to have 13 marbles altogether? (*Change unknown*)



Roy has some marbles. Debbie gave him 5 more marbles. Now he has 13 marbles. How many marbles did Roy have to start with? (*Start unknown*)



Abstract

Links between concrete approach and written method are made explicit at each stage.

$ \begin{array}{r} \text{T} \quad \text{O} \\ 1 \quad 5 \\ + \quad 1 \quad 8 \\ \hline 1 \quad 3 \\ 2 \quad 0 \\ \hline 3 \quad 3 \end{array} $	→	$ \begin{array}{r} \text{T} \quad \text{O} \\ 1 \quad 5 \\ + \quad 1 \quad 8 \\ \hline 3 \quad 3 \\ \hline 1 \end{array} $
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Talk through the method, making the links to the concrete method very clear to explain the stages of the calculation.

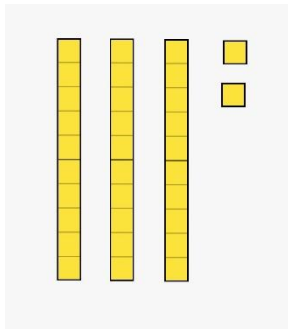
Year 2 Subtraction

Concrete

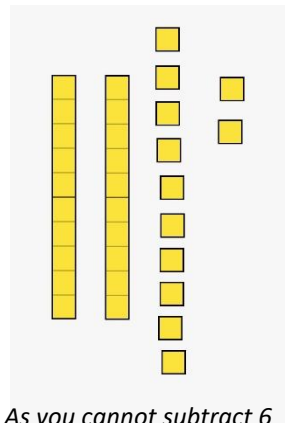
Continue to use resources introduced in Year 1.

Increased use of Base 10 materials to show the stages in a calculation requiring regrouping and renaming.

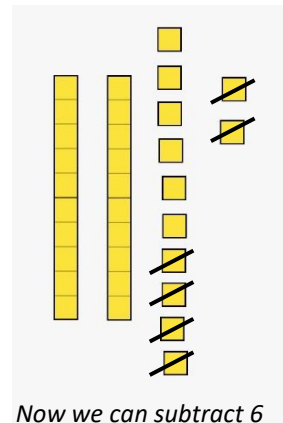
$$32 - 16 = \square$$



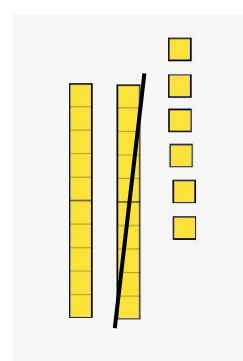
Start with 32



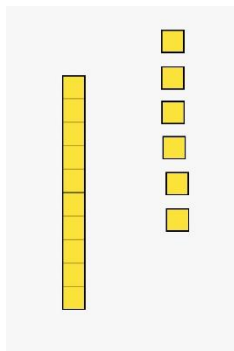
As you cannot subtract 6 ones from 2 ones we need to regroup one of the tens and rename it ten ones.



Now we can subtract 6 ones from the 12 ones.



Then subtract one ten from the two tens.



Leaving the answer 16.

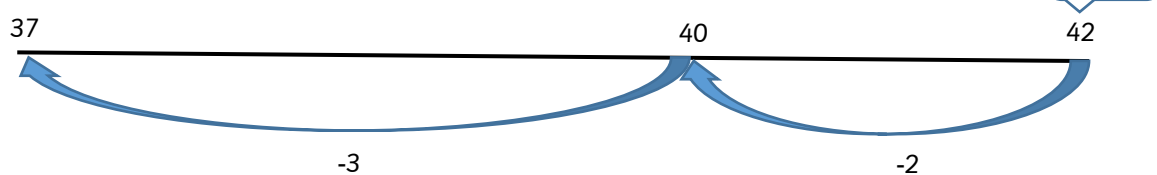
$$32 - 16 = 16$$

Pictorial

Base 10 methods 'illustrated' in pupil diagrams.

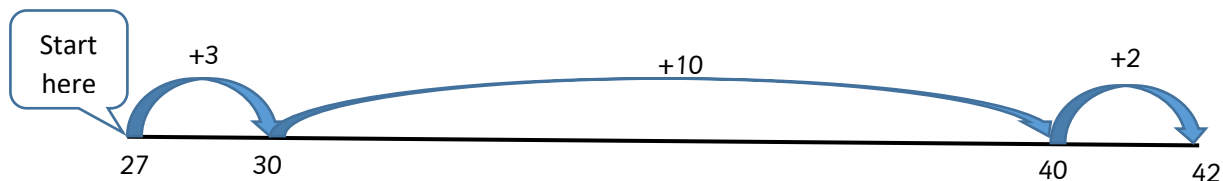
Extend use of number lines with larger "jumps" (including crossing the tens)

$$42 - 5 = \square$$



Use of number lines with 'find the difference' questions.

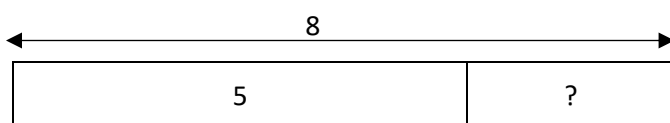
$$42 - 27 = \square$$



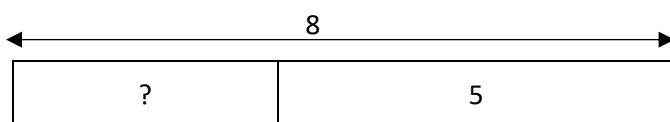
An introduction to bar modelling for subtraction in a variety of formats:

Part-Part-Whole Problems: Part Unknown

$$8 - 5 = \square$$

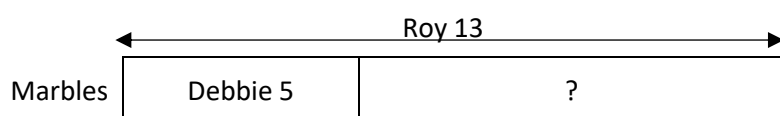


$$8 - \square = 5$$

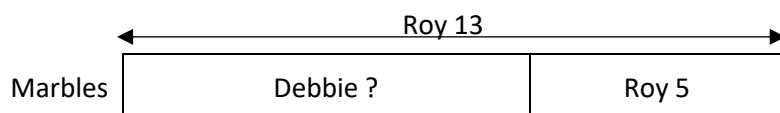


Extending to context based problems such as:

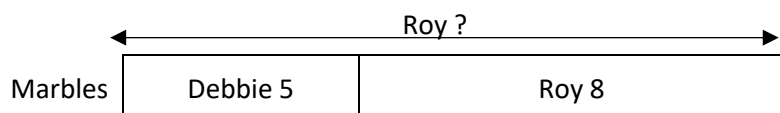
Roy has 13 marbles. He gave 5 to Debbie. How many marbles does Roy have left? (*Result unknown*)



Roy had 13 marbles. He gave some to Debbie. Now he has 5 marbles left. How many marbles did Roy give to Debbie? (*Change unknown*)

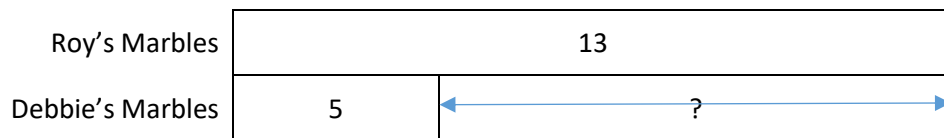


Roy had some marbles. He gave 5 to Debbie. Now he had 8 marbles left. How many marbles did Roy have to start with? (*Start unknown*)

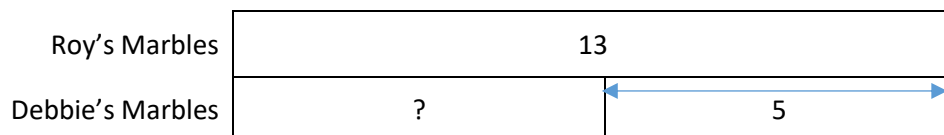


The bar models will also be used for comparing problems:

Roy has 13 marbles. Debbie has 5 marbles. How many more marbles does Roy have than Debbie?
(*Difference unknown*)



Roy has 13 marbles. He has 5 more marbles than Debbie. How many marbles does Debbie have?
(*Smallest part unknown*)



Debbie has 5 marbles. Roy has 8 more than Debbie. How many marbles does Roy have?
(*Largest part unknown*)



Abstract

Links between concrete approach and written method are made explicit at each stage.

Formal calculations are initially taught with no need for regrouping and renaming

$$\begin{array}{r}
 \text{T O} \\
 34 \\
 - 21 \\
 \hline
 13
 \end{array}$$



$$\begin{array}{r}
 \text{T O} \\
 \cancel{23}^{12} \\
 - 16 \\
 \hline
 16
 \end{array}$$

The calculations are “talked through” with the reasoning being made explicit

Year 3

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

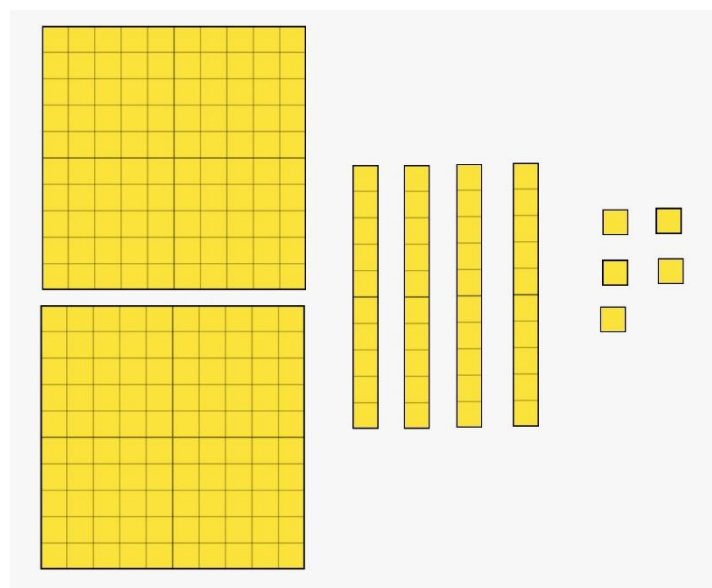
- add and subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 3 Addition

Concrete

As new concepts are taught, reference continues to be made to the concrete and visual representations from earlier in the school to reinforce the reasoning behind the calculations.

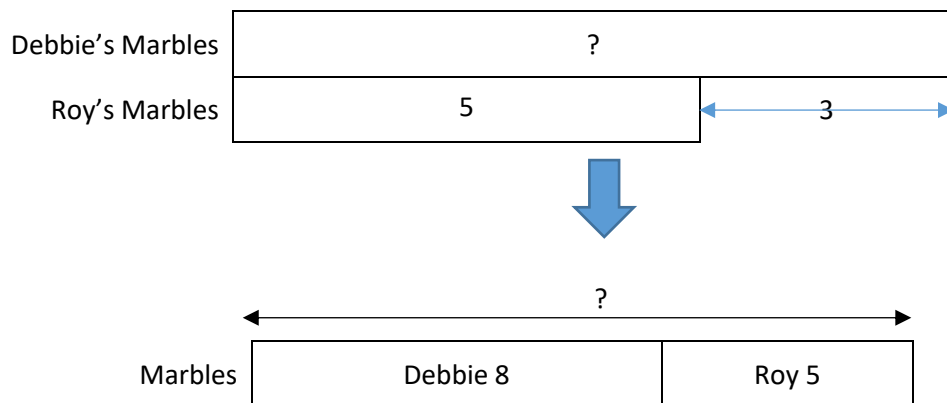
Base 10 materials continue to be used heavily to represent number.



Pictorial

The mastery of additive reason problems from KS1 is checked, with revision as necessary. Develop use of bar model with two step joining problems:

Roy has 5 marbles. Debbie has 3 more than Roy. How many marbles do they both have altogether?



Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation.

<table border="0" style="margin-left: auto; margin-right: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>2</td><td>7</td><td>8</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td>8</td><td>2</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td>1</td><td>0</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>1</td><td>5</td><td>0</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>2</td><td>0</td><td>0</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>3</td><td>6</td><td>0</td></tr> <tr><td colspan="3"><hr/></td></tr> </table>	H	T	O	2	7	8	<hr/>				8	2	<hr/>				1	0	<hr/>			1	5	0	<hr/>			2	0	0	<hr/>			3	6	0	<hr/>			➔	<table border="0" style="margin-left: auto; margin-right: auto;"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>2</td><td>7</td><td>8</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td>8</td><td>2</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>3</td><td>6</td><td>0</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>1</td><td>1</td><td></td></tr> </table> <div style="border: 1px solid black; padding: 5px; margin-top: 10px; width: fit-content;"> <p>Cross off the number after it has been added to the other numbers in that column</p> </div>	H	T	O	2	7	8	<hr/>				8	2	<hr/>			3	6	0	<hr/>			1	1	
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Year 3 Subtraction

Concrete

As new concepts are taught, reference continues to be made to the concrete and visual representations from earlier in the school to reinforce the reasoning behind the calculations.

Base 10 materials continue to be used heavily to represent numbers in the calculations.

Pictorial

Full understanding of bar modelling from KS1 is checked and revised as necessary.

Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation, beginning with three digit columnar written strategies. Initially with no exchange, then with the regrouping of tens into ones (as initially introduced in Year 2).

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 5 \quad \cancel{2} \quad ^12 \\ - 3 \quad 1 \quad 6 \\ \hline 2 \quad 1 \quad 6 \end{array}$$

Year 4

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Year 4 Addition

Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation moving into 4 digit calculations.

$$\begin{array}{r} \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\ 7 \quad 3 \quad 1 \quad 0 \\ + \quad 5 \quad 9 \quad 3 \quad 8 \\ \hline 1 \quad 3 \quad 2 \quad 4 \quad 8 \\ \hline \end{array}$$

✓ ✓

Begin to explore decimals in the context of money

$$\begin{array}{r} \text{T} \quad \text{O} \cdot \frac{1}{10} \quad \frac{1}{100} \\ \pounds \quad 1 \quad 8 \cdot 7 \quad 2 \\ + \quad \pounds \quad 5 \quad 9 \cdot 2 \quad 9 \\ \hline \pounds \quad 7 \quad 8 \cdot 0 \quad 1 \\ \hline \end{array}$$

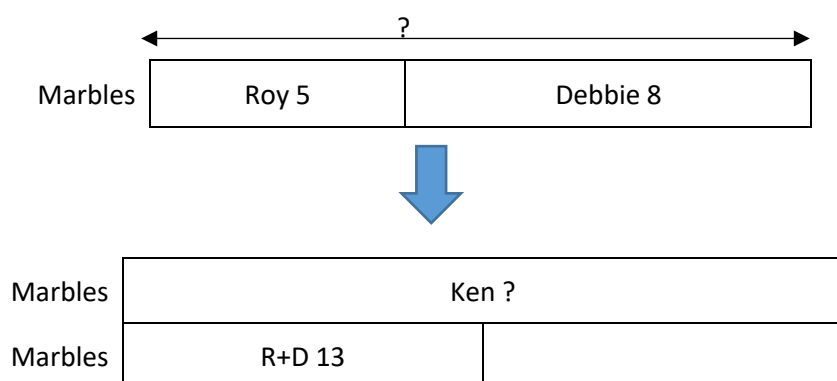
✓ ✓ ✓

Year 4 Subtraction

Pictorial

Develop repertoire of bar modelling to include 2 step separating problems:

Roy has 5 marbles and Debbie has 8 marbles. Ken says, "I have double the number of marbles you have together." How many marbles does Ken have?



Abstract

As before, continue to model and reinforce with concrete resources and visual representations throughout.

Extend to subtraction of 4 digit numbers with exchanging (regrouping and renaming) now introduced from hundreds into tens, building to thousands into hundreds. Initially, questions will be asked where only one exchange is needed.

$$\begin{array}{r}
 \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\
 5 \quad \cancel{2} \quad 13 \quad 9 \\
 - 3 \quad 1 \quad 4 \quad 6 \\
 \hline
 2 \quad 1 \quad 9 \quad 3
 \end{array}$$

Begin to explore decimals in the context of money

$$\begin{array}{r}
 \text{T} \quad \text{O} \cdot \frac{1}{10} \quad \frac{1}{100} \\
 \pounds \quad 9 \quad \cancel{0} \cdot 18 \quad 5 \\
 - \pounds \quad 5 \quad 0 \cdot 9 \quad 1 \\
 \hline
 \pounds \quad 4 \quad 0 \cdot 9 \quad 4
 \end{array}$$

Year 5

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Year 5 Addition

Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent. Strategies build on those of Year 4 and involve starting with numbers up to 100,000 and progressing to 1,000,000.

	HTh	TTh	Th	H	T	O
	7	1	2	3	1	0
	8	3	4	1	6	3
+	5	3	2	9	3	8
<hr/>						
	2	0	7	9	4	1
	1	0	7	9	4	1
<hr/>						
	2					
	2					

Progress to addition of numbers to two decimal places in context (such as money or measurement).

	H	T	O	$\cdot \frac{1}{10}$	$\frac{1}{100}$
	£	5	1	8	· 7 2
+	£	3	5	9	· 2 9
<hr/>					
	£	8	7	8	· 0 1
<hr/>					
				1	1 1

As throughout, consider the appropriateness of the numbers, initially starting with one carry to ensure clarity and understanding of the layout and process before gradually increasing the complexity of the calculations.

Year 5 Subtraction

Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent.

Strategies build on those of Year 4 and involve starting with numbers up to 100,000 and progressing to 1,000,000.

Progressively, and before moving to larger numbers, begin to explore written strategies where '2 exchanges' are needed.

	Th	H	T	O
	7	8 ⁹	10 ⁹	16
-	2	5	9	8
<hr/>				
	5	3	0	8
<hr/>				

Year 6

Number – addition and subtraction

Statutory requirements

Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Year 6 Addition

Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent. Strategies build on those of Year 4 and Year 5 with numbers within 10 million and calculations with up to 3 decimal places (through problems set in contextual situations, such as measurement).

Year 6 Subtraction

Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent. Strategies build on those of Year 4 and Year 5 with numbers within 10 million and calculations with up to 3 decimal places (through problems set in contextual situations, such as measurement).

$$\begin{array}{r} \overset{2}{\cancel{2}} \overset{1}{5} \cdot \overset{6}{\cancel{7}} \overset{10}{\cancel{1}} 12 \\ - 8 \cdot 6 5 3 \\ \hline 2 7 \cdot 0 5 9 \end{array}$$

MULTIPLICATION & DIVISION

Year 1

Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 1 Multiplication

Concrete

The use of lots of use of physical objects to model the process of multiplication.



Developed into the arrangement of these displayed as arrays.

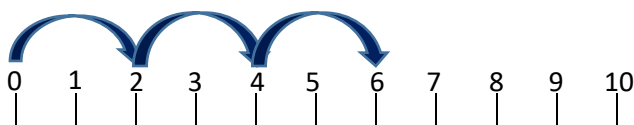


Pictorial

Pupils make jottings of the problems and the arrays.

Links are made with repeated addition.

$$3 \times 2 = 6$$



Year 1 Division

Concrete

The use of lots of use of physical objects to model the process of division.

The key difference between 'grouping' and 'sharing' in the context of division problems is explicitly modelled.



$6 \div 2 = 3$ can be demonstrated by "grabbing" groups of 2 frogs and finding out how many lily pads are needed.



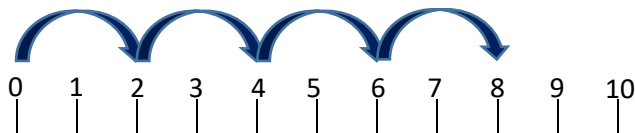
$6 \div 2 = 3$ can be demonstrated sharing the 6 frogs equally between 2 lily pads.

Pictorial

Pupils make jottings of the various problems and the arrays.

Links are made with repeated addition.

$8 \div 2 =$ "How many 2s make 8?"



Year 2

Number – multiplication and division

Statutory requirements

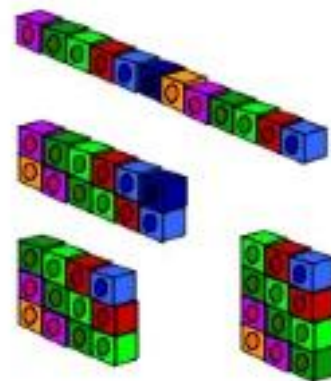
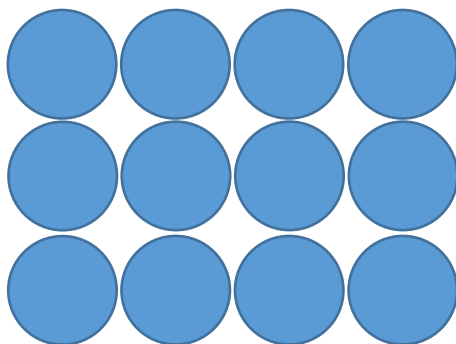
Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 2 Multiplication

Concrete

Develop the work from Year 1 on the use of arrays, using counters/cubes to show multiplication sentences.

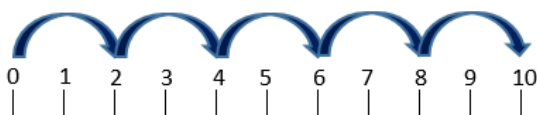


Pictorial

Develop informal jottings to support the calculation.

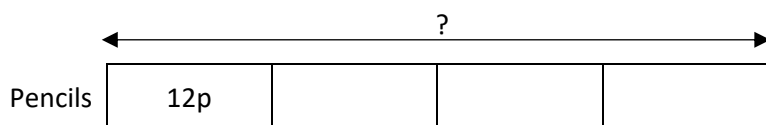
Build on the number line representation from Year 1.

2 x 5 is the same as 5 x 2



Bar modelling is introduced for multiplicative reasoning with part-whole problems:

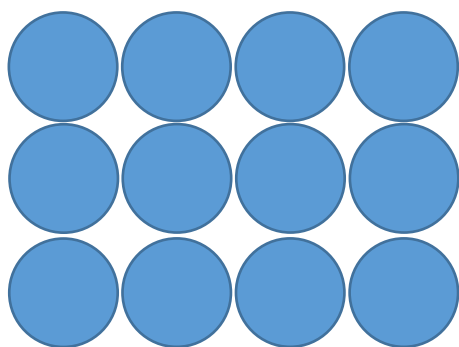
Pencils cost 12p each. How much do 4 pencils cost? (*Whole unknown*)



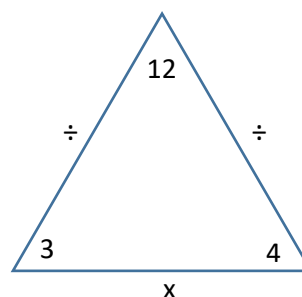
Year 2 Division

Concrete

With the use of counters and cubes, the link between multiplication and division is made explicit by creating an array and generating the different number sentences that can be created.

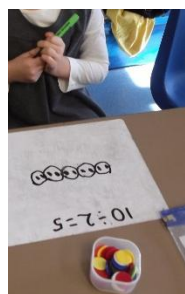


$4 + 4 + 4 = 12$	$3 \times 4 = 12$
$3 + 3 + 3 + 3 = 12$	$4 \times 3 = 12$
$12 - 4 - 4 - 4 = 0$	$12 \div 4 = 3$
$12 - 3 - 3 - 3 - 3 = 0$	$12 \div 3 = 4$

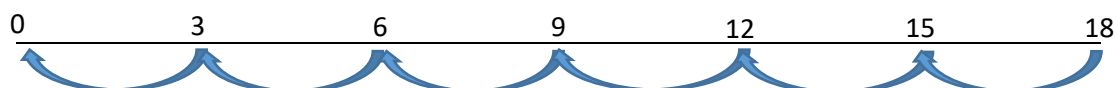


Pictorial

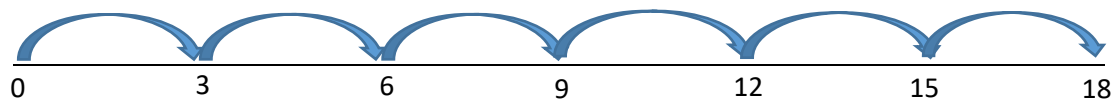
Pupils use jottings to express the division problems.



$18 \div 3$ can be modelled as sharing – 18 divided between 3 or by modelling jumping back in threes to share in 'chunks' of 3



It can also be modelled in terms of grouping – how many 3s make 18?



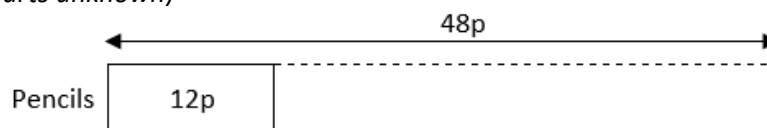
(In all calculations up to this point there are no remainders and the problems are built upon multiplication facts that the pupils are expected to be fluent in).

Bar Modelling is introduced for multiplicative (and the related divisional) reasoning:

Barry bought 4 pencils for 48p. How much does 1 pencil cost? (*Value of one part unknown*)



Pencils cost 12p each. David bought some pencils for 48p. How many pencils did he buy? (*Number of parts unknown*)



Year 3

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

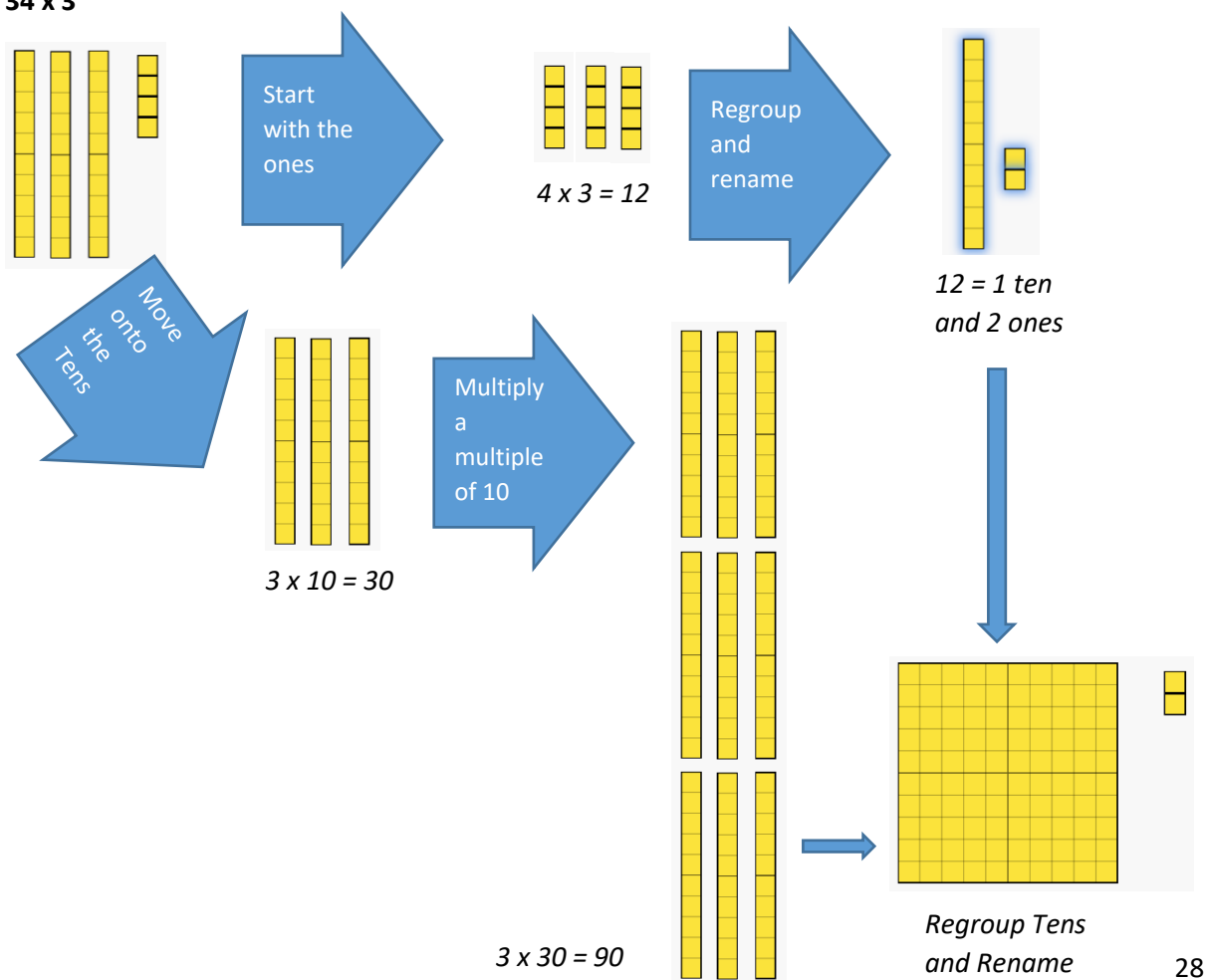
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 3 Multiplication

Concrete

Base 10 materials are heavily used to model the multiplication of number.

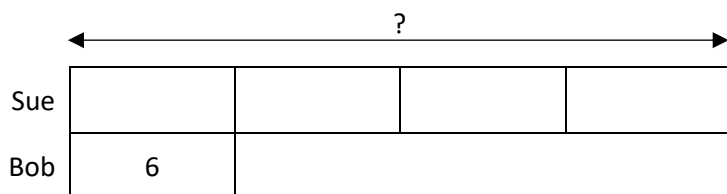
34 x 3



Pictorial

The mastery of multiplicative reasoning problems from KS1 is checked, with revision as necessary. Develop use of bar model with comparing problems:

Bob picked 6 apples. Sue picked four times as many apples as Bob. How many apples did Sue pick? (*Larger quantity unknown*)



Abstract

Very close links are made between the concrete method (above) and the initial steps towards the formal written method, developed further in Year 4. Calculations are performed side by side.

$$34 \times 3$$

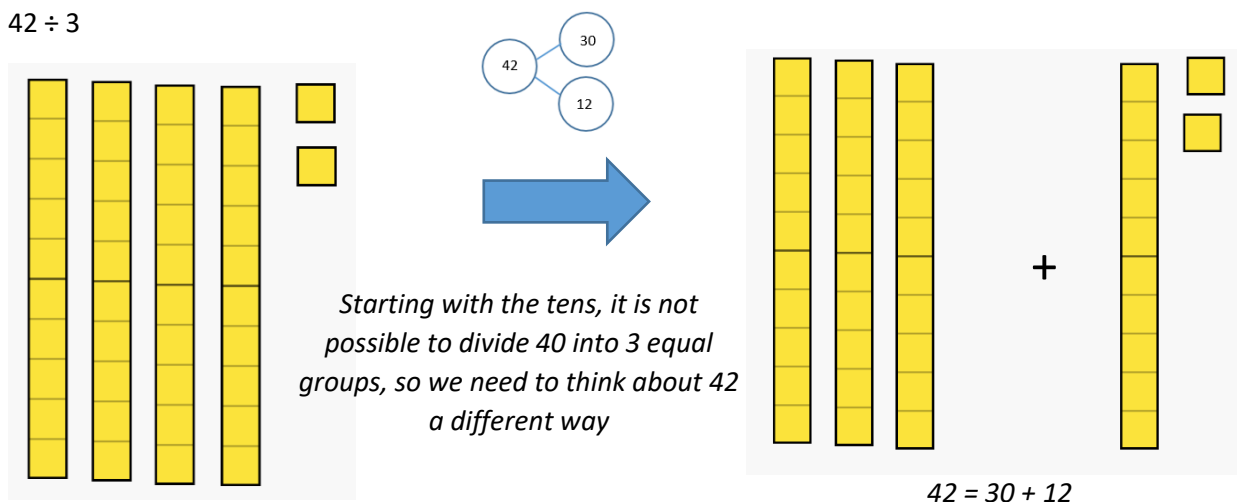
					H	T	O
	4	x	3	=		1	2
+	3	0	x	3	=	9	0
						1	0
						2	

Year 3 Division

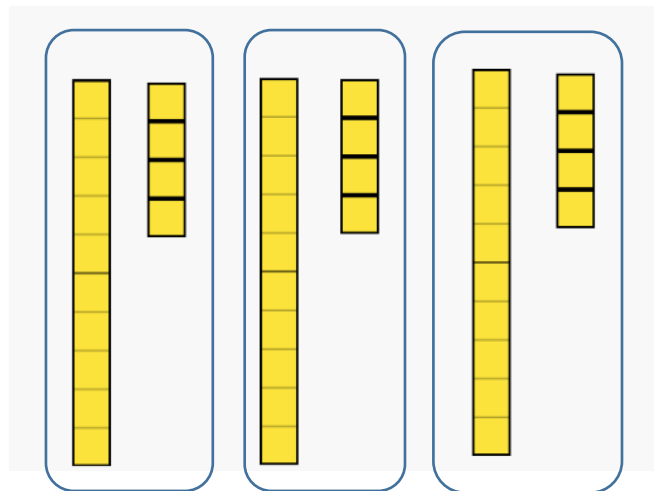
Concrete

The concrete resources from lower down the school continue to be used to demonstrate the difference between grouping and sharing. As with multiplication, extensive use is made of Base 10 materials to demonstrate what is happening with the division.

$$42 \div 3$$



The tens can now be divided into 3 equal groups. The 12 ones can also be equally divided into 3 groups:

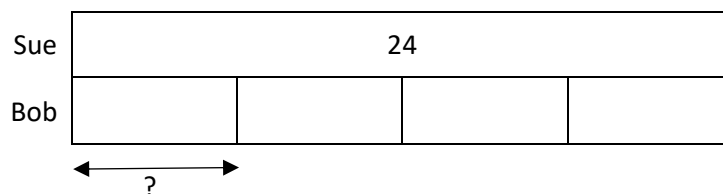


So 42 divided by 3 results in 3 equal groups of 14.

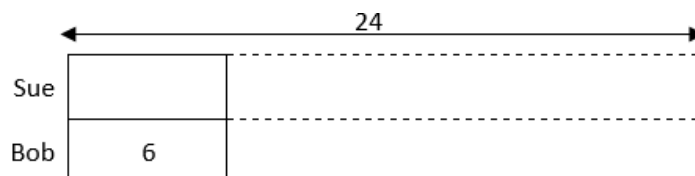
Pictorial

The mastery of multiplicative (and related divisional) reasoning problems from KS1 is checked, with revision as necessary. Develop use of bar model with comparing problems:

Sue picked 24 apples. She picked four times as many apples as Bob. How many apples did Bob pick? (*Smaller quantity unknown*)



Sue picked 24 apples. Bob picked 6 apples. How many more times as many apples did Sue pick than Bob? (*Multiplier unknown*)



Abstract

With the links between the concrete representation (using Base 10 materials) very clear, modelling the process side by side.

$$\begin{array}{r} \quad\quad 1\quad 4 \\ 3\overline{\Big|} \quad 4\quad 2 \\ -\quad 3\quad 0\quad (10x) \\ \hline \quad 1\quad 2 \\ -\quad 1\quad 2\quad (4x) \\ \hline \quad 0 \end{array}$$

Year 4

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Year 4 Multiplication

Pictorial

Bar Model approaches introduced in KS1 and Year 3 for multiplicative reasoning (and related divisional problems) are used and revised as necessary.

Abstract

Continue to model concrete and visual representations practically alongside the written calculations which are more formally introduced in Year 4.

Calculations follow the 'expanded' formal written method:

	H	T	O	
	1	4	3	
x			6	
<hr/>				
	1	8		(3 x 6)
	2	4	0	(40 x 6)
	6	0	0	(100 x 6)
<hr/>				
	8	5	8	
<hr/>				

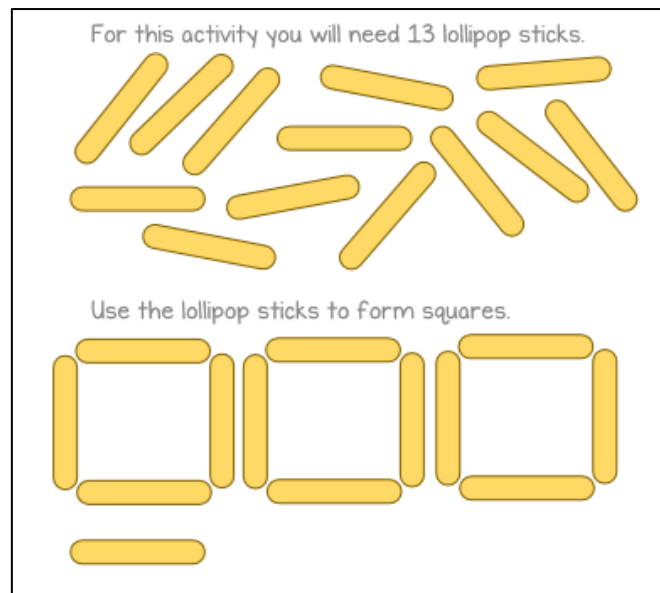
Derived from the knowledge that as $4 \times 6 = 24$ then 40×6 would be 10 times greater.

Year 4 Division

Concrete

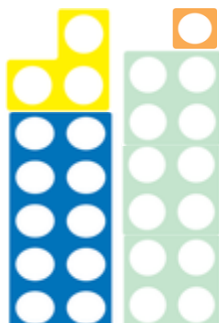
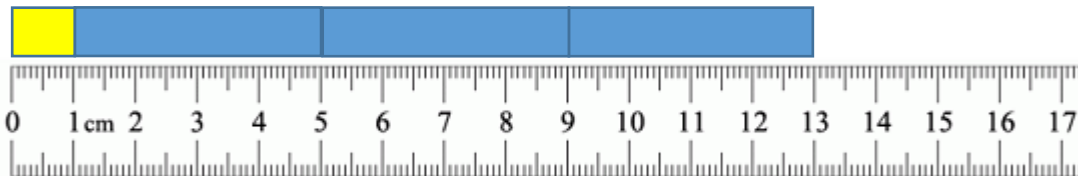
As the introduction of the concept of remainders with division is introduced, there is an increased use of concrete resources to reinforce this learning.

For example, using lollipop sticks to investigate a sum such as $13 \div 4$



$13 \div 4$ can be represented as 13 lollipop sticks make 4 squares with one stick left over ($13 \div 4 = 3 \text{ r}1$)

This would also be modelled with counters/objects as lower in the school as well as



Pictorial

Bar Model approaches introduced in KS1 and Year 3 for multiplicative reasoning (and related divisional problems) are used and revised as necessary.

Abstract

$$432 \div 5 =$$

		8	6	r2
5		4	3	2
-		4	0	0 (80x)
		<hr/>		
		3	2	
-		3	0	(6x)
		<hr/>		
			2	
		<hr/>		

Pupils encouraged to accompany this method with jottings:

5
10
15
20
25
30
35
40
45
50

Year 5

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Year 5 Multiplication

Abstract

Continue to model concrete and visual representations practically alongside the written calculations.

	H	T	O
	1	4	3
x			6
<hr/>			
	8	5	8
<hr/>			
	2	1	

Developing later in the year to 2 digit by 2 digit calculations.

		T	O
		3	7
x		2	9
<hr/>			
	3	3	3
<hr/>			
	3	6	
	7	4	0
<hr/>			
	1		
1	0	7	3
<hr/>			
	1		

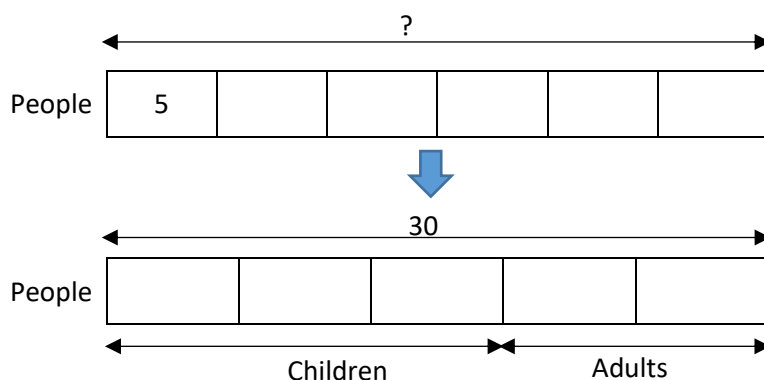
The zero is added at the beginning to remind that the calculation is $\times 20$ (therefore 10 times bigger than $\times 2$)

Year 5 Division

Pictorial

Check on retention of use of bar modelling for part-part-whole and comparing problems from Year 3 and Year 4, revising as necessary. Develop use of bar models with multi-step part-whole problems:

There are 5 people living in each of the 6 houses on Green Street. $\frac{3}{5}$ of these people are children and the rest are adults. How many adults live on Green Street?



Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent.

Pupils apply the short division strategy (bus stop method)

$$\begin{array}{r} 196 \\ 6 \overline{) 1176} \end{array}$$

Extend this to deal with a remainder:

$$\begin{array}{r} 196 \text{ r}3 \\ 6 \overline{) 1176} \end{array}$$

Pupils continue to make jottings to help with the calculations:

6
12
18
24
30
36
42
48
54
60

Extend further to interpret the remainder as a fraction ($\frac{3}{6}$) and then as a decimal:

$$\begin{array}{r} 196.5 \\ 6 \overline{) 1176.30} \end{array}$$

Year 6

Number – multiplication and division

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

Year 6 Multiplication

Abstract

Continue to model concrete and visual representations practically alongside the written calculations.

2314 x 23

	Th	H	T	O	
	2	3	1	4	
x			2	3	
<hr/>					
	6	9	4	2	(x3)
			<i>1</i>		
4	6	2	8	0	(x20)
<hr/>					
5	3	2	2	2	
<i>1</i>	<i>1</i>	<i>1</i>			

Progress onto calculations involving decimals:

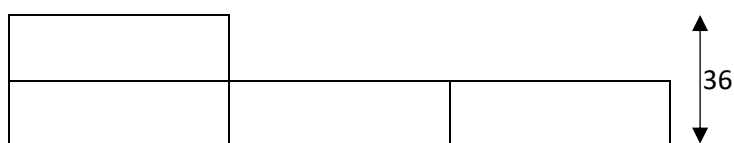
$$\begin{array}{r}
 \begin{array}{cccc}
 & H & T & O & \frac{1}{10} \\
 & & 3 & 6 & \cdot 2 \\
 \times & & & 7 & \\
 \hline
 & 2 & 5 & 3 & \cdot 4 \\
 \hline
 & \cancel{2} & \cancel{4} & \cancel{1} &
 \end{array}
 \end{array}$$

Year 6 Division

Pictorial

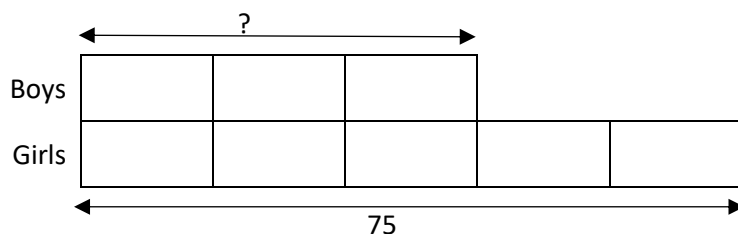
Bar modelling continues to be used to solve problems, including comparing problems:

The sum of two numbers is 36. The larger number is 3 times the smaller number. What are the two numbers?



(The four equal divisions must add up to 36 and therefore each box should contain 9. The two numbers are 9 and 27).

There are $\frac{3}{5}$ as many boys as girls. If there are 75 girls, how many boys are there?



(The five equal divisions for the girls result in each box being worth 15. Therefore the 3 boxes for the boys add up to 45).

There is a clear link between these bar modelling approaches and how ratio and proportion can be modelled effectively with the bar approach.

Abstract

Revisit the bus stop method to ensure full understanding.

Extend the “chunking” method initially introduced in Year 4 for larger calculations.

$$\begin{array}{r}
 \begin{array}{cccc} & 2 & 6 & 5 & r7 \end{array} \\
 1 \quad 5 \quad \overline{) \begin{array}{cccc} 3 & 9 & 8 & 2 \end{array}} \\
 \underline{\begin{array}{cccc} 3 & 0 & 0 & 0 \end{array}} & (200x) \\
 \begin{array}{cccc} & 9 & 8 & 2 \end{array} \\
 \underline{\begin{array}{cccc} & 7 & 5 & 0 \end{array}} & (50x) \\
 \begin{array}{cccc} & & 12 & 2 \end{array} \\
 \underline{\begin{array}{cccc} & & 10 & 0 \end{array}} & (10x) \\
 \begin{array}{cccc} & & & 2 \end{array} \\
 \underline{\begin{array}{cccc} & & & 0 \end{array}} & (5x) \\
 \begin{array}{cccc} & & & 2 \end{array} \\
 \underline{\begin{array}{cccc} & & & 0 \end{array}} \\
 \begin{array}{cccc} & & & 2 \end{array}
 \end{array}$$

Jottings are used to support pupils in the calculation:

x1	15	x10	150	x100	1500
x2	30	x20	300	x200	3000
x5	75	x50	750	x500	7500

To work out x5
halve x10

As in Year 5, extend understanding of the remainder as a fraction and also as a decimal.