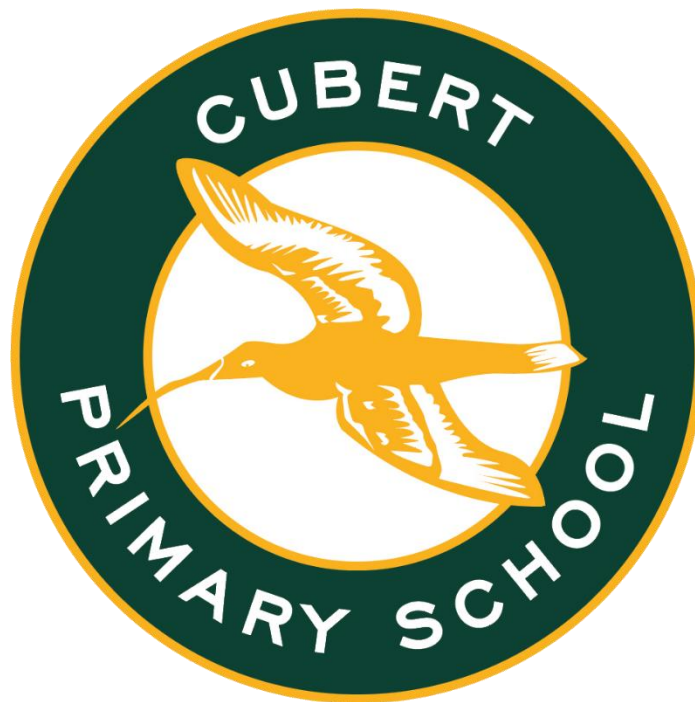
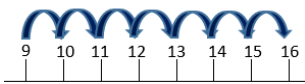
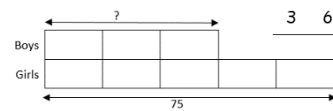


	H	T	O
	2	7	8
+		8	2
		1	0
	1	5	0
	2	0	0
	3	6	0



# Calculation Policy

## Introduction

This calculation policy is designed to ensure consistency and progression in the teaching of addition, subtraction, multiplication and division across the school. It is aligned with the 2014 National Curriculum.

Children will use mental calculation approaches as their first port of call when it is efficient and appropriate to do so. When necessary, an efficient written method needs to be used accurately, confidently and with clear understanding.

Within each section there are examples of concrete (the practical items that pupils can hold and manipulate to help them explore abstract mathematical concepts and the relationships between them), pictorial (models and representations) and abstract (the symbolic stage).

### **Concrete**

Concrete is the “doing” stage, using concrete objects to model problems. Instead of the traditional method of maths teaching, where a teacher demonstrates how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical objects themselves. Every new abstract concept is learned first with a “concrete” or physical experience.

For example, if a problem is about adding up four baskets of fruit, the children might first handle actual fruit before progressing to handling counters or cubes which are used to represent the fruit.

### **Pictorial**

Pictorial is the “seeing” stage, using representations of the objects to model problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem.

Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult, such as fractions, as it helps them visualise the problem and make it more accessible.

### **Abstract**

Abstract is the “symbolic” stage, where children are able to use abstract symbols to model problems (Hauser).

Only once a child has demonstrated that they have a solid understanding of the “concrete” and “pictorial” representations of the problem, can the teacher introduce the more “abstract” concept, such as mathematical symbols. Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols, for example  $+$ ,  $-$ ,  $\times$ ,  $\div$  to indicate addition, multiplication, or division.

Although presented as three distinct stages, it would be expected for teaching to go back and forth between each representation to reinforce concepts.

This document also contains exemplars from the White Rose Calculation Policy which details the specific representations and question stems for each of the skills in the 4 rules.

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# ADDITION & SUBTRACTION

## EYFS

*'Developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop a deep conceptual understanding of the numbers to 10, the relationships between them and the patterns therein. By providing frequent and varied opportunities to build and apply this understanding – such as using manipulatives – children will develop a secure base of knowledge from which mathematical mastery is built. In addition, children's curiosity about number, shape, space and measure should be encouraged and furthered through opportunities to apply their growing understanding of the mathematical world to the world around them.'*

<b>Area</b>	<b>Early Learning Goals (ELG)</b>
<i>Numbers</i>	<ul style="list-style-type: none"><li>*Have a deep understanding of number to 10, including the composition of each number.</li> <li>*Subitise (recognise quantities without counting) up to 5.</li> <li>*Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.</li></ul>
<i>Numerical Patterns</i>	<ul style="list-style-type: none"><li>*Verbally count beyond 20, recognising the pattern of the counting system.</li> <li>*Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity.</li> <li>*Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li></ul>

## EYFS Addition

(Including Nursery to track back the path of progression)





# Addition

<b>Nursery</b>	<ul style="list-style-type: none"> <li>Begin to have an understanding of numbers to 5</li> <li>We recommend focusing on noticing and representing small quantities, perceptual subitising and counting.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<b>Subitise to 3</b> Instantly see how many.	How many do you see? 	
<b>Count how many</b> Begin to count objects using 1-1 correspondence.	How many are there? 1 2 3 4 5 	Count out ... from a larger group. E.g. Collect 3 beanbags for a game. 
<b>Make numbers to 5</b> Start by showing 1, 2 and 3 using fingers.	Show me... 	Begin to link numerals to quantities. 
<b>Add 1 more</b> Through stories, songs and rhymes.	How many do I have now? 	

# Addition







<b>Reception</b>	<ul style="list-style-type: none"> <li>Have a deep understanding of numbers to 10, including the composition of each number.</li> <li>Subitise (recognise quantities without counting) up to 5</li> <li>Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<b>Conceptually subitise to 5</b> Notice the parts that make up the whole.	What do you see? How do you see it? 	
<b>1 more</b> Continue to link to stories, songs and rhymes.	1 more than ... is ... 	
<b>Notice the composition of numbers within 10</b> Link to stories, songs and rhymes.	How many...? How many...? How many altogether? 	How many ways can you make...? 

# Addition

Progression of skills	Key representations	
<p><b>Combine 2 groups</b></p> <p>2 groups are combined to find the total.</p>	<p>There are .... There are .... There are .... altogether.</p> 	<p>.... and .... make ....</p> 
<p><b>Add more</b></p> <p>A quantity is increased.</p>	<p>First... Then.... Now....</p> 	<p>I have .... I add .... more. Now I have....</p> 




## EYFS Subtraction

# Subtraction


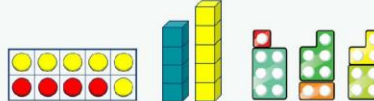
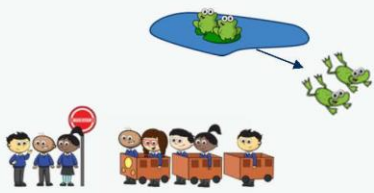
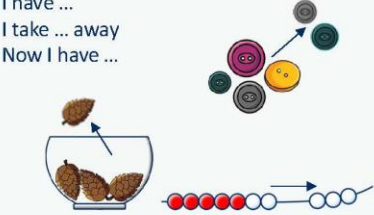
<b>Nursery</b>	<ul style="list-style-type: none"> <li>Begin to have an understanding of numbers to 5</li> <li>We recommend focusing on noticing and representing small quantities, perceptual subitising and counting.</li> </ul>	
Progression of skills	Key representations	
<p><b>Subitise to 3</b></p> <p>Instantly see how many.</p>	<p>How many do you see?</p> 	
<p><b>Count how many</b></p> <p>Begin to count objects using 1-1 correspondence.</p>	<p>How many are there?</p> <p>1 2 3 4 5</p> 	<p>Count out ... from a larger group. E.g. Collect a cup for everyone at the table.</p> 
<p><b>Make numbers to 5</b></p> <p>Start by showing 1, 2 and 3 using fingers.</p>	<p>Show me...</p> 	<p>Begin to link numerals to quantities.</p> 
<p><b>Take 1 away</b></p> <p>Through stories, songs and rhymes.</p>	<p>How many do we have now?</p> 	



# Subtraction

<p><b>Reception</b></p>	<ul style="list-style-type: none"> <li>Have a deep understanding of number to 10, including the composition of each number.</li> <li>Subitise (recognise quantities without counting) up to 5</li> <li>Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (and some subtraction facts) and some number bonds to 10, including double facts.</li> </ul>
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Conceptually subitise to 5</b></p> <p>Notice the parts that make up the whole.</p>	<p>What do you see? How do you see it?</p> 
<p><b>1 less</b></p> <p>Continue to link to stories, songs and rhymes.</p>	<p>1 less than ... is ...</p> 
<p><b>Notice the composition of numbers within 10</b></p> <p>Link to stories, songs and rhymes.</p>	<p>How many...? How many...? How many altogether?</p> <p>How many ways can you make...?</p> 

# Subtraction

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Partition</b></p> <p>Using objects, explore different ways to partition a number into 2 or more parts.</p>	<p>There are ... altogether. I can see ... here and ... there.</p> 	<p>... and ... make ...</p> 
<p><b>Take away</b></p> <p>A quantity is reduced.</p>	<p>First... Then... Now...</p> 	<p>I have ... I take ... away Now I have ...</p> 

# Year 1

## Number – addition and subtraction

### Statutory requirements

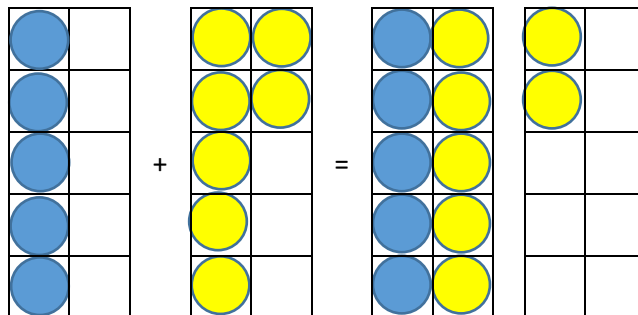
Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

### Year 1 Addition

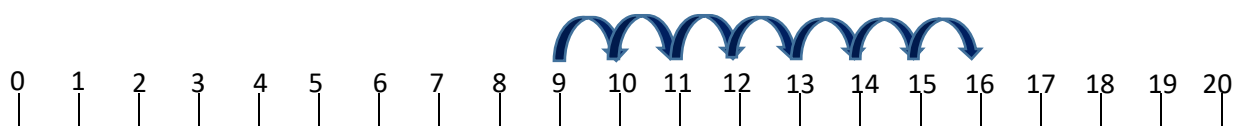
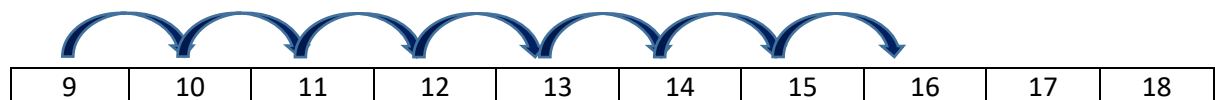
#### Concrete

Ten Frames are used to develop the ‘sense of number’ and identify the number bonds within 10 and up to 20, a great deal of use is made of 10 frames, looking at patterns within the numbers. This leads into discussion of how to look at an addition such as  $5 + 7$ .



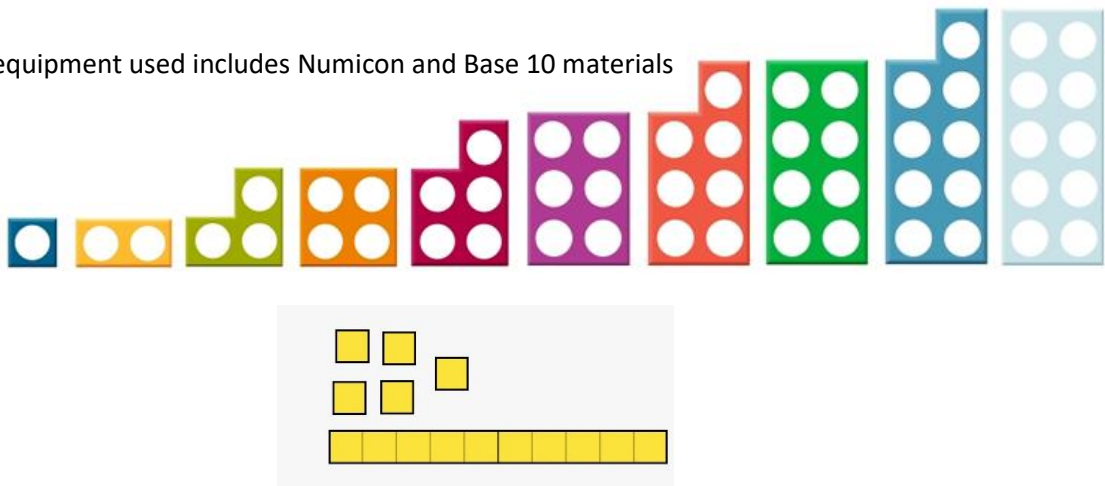
Pupils count on with number tracks/number lines

$$9 + 7 = \square$$



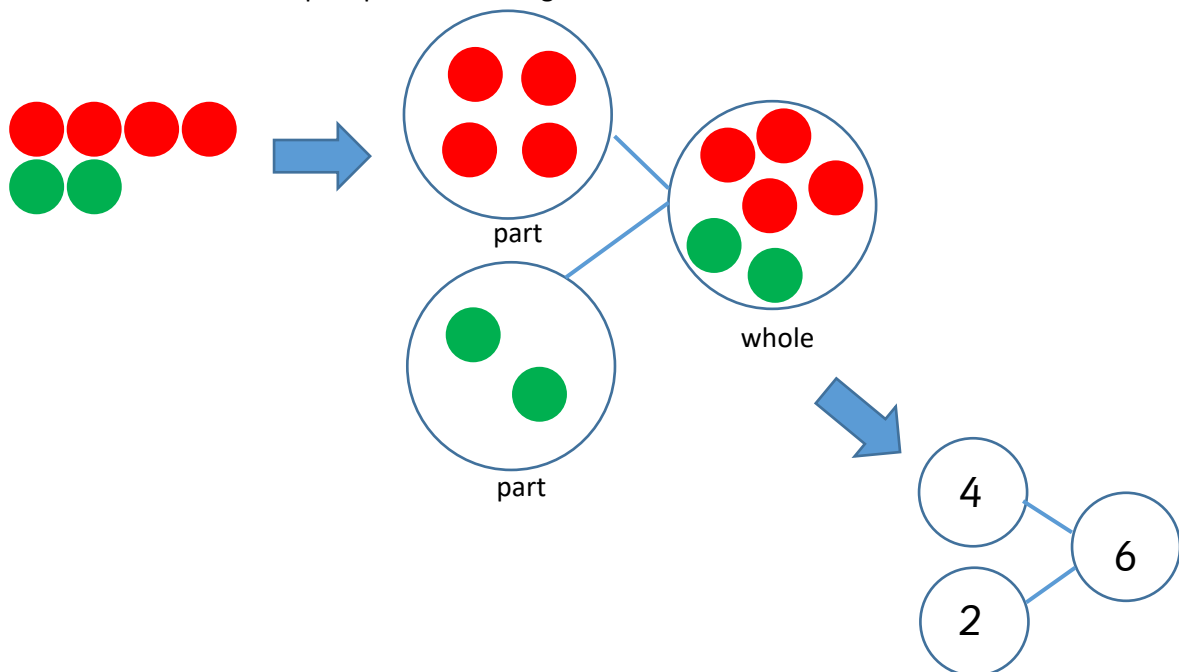
Lots of activities take place with physical resources (counters, cubes, coins, toy characters etc.)

Other equipment used includes Numicon and Base 10 materials



### Pictorial

Extensive use is made of part-part-whole Diagrams:

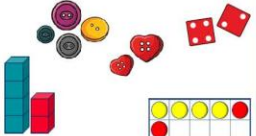
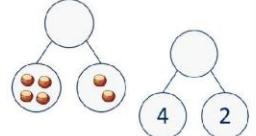
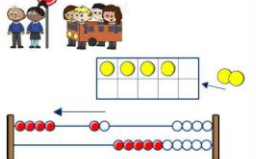
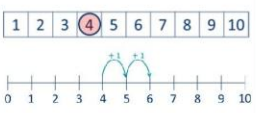


### Abstract

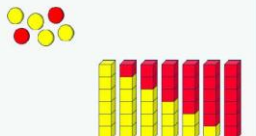
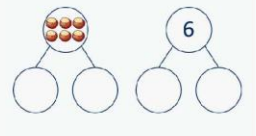
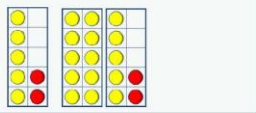
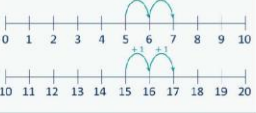

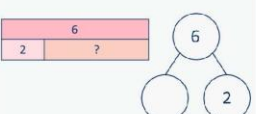

The introduction of addition stories.

Introduction of  $\square = 9 + 7$  to demonstrate the meaning of the equals sign in balanced equations (reinforced with practical examples using scales) and commutative number sentences showing that  $4+2$  is the same as  $2+4$ .

# Addition

<b>Year 1</b>	<ul style="list-style-type: none"> <li>Read, write and interpret mathematical statements involving addition (+) and equals (=) signs.</li> <li>Represent and use number bonds within 20</li> <li>Add 1-digit and 2-digit numbers to 20, including zero.</li> <li>Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as <math>7 = \square + 2</math></li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Add together</b> (aggregation)  2 quantities are combined to find the total.	There are ... There are ... There are ... altogether.  	... is a part. ... is a part. ... is the whole.  	... plus ... is equal to ... ... is equal to ... + ...  $4 + 2 = 6$ $2 + 4 = 6$  $6 = 4 + 2$ $6 = 2 + 4$
<b>Add more</b> (augmentation)  A quantity is increased.	First... Then... Now...  	I start at ... I jump on ... I land on ...  	... plus ... is equal to ... ... is equal to ... + ...  $4 + 2 = 6$ $2 + 4 = 6$  $6 = 4 + 2$ $6 = 2 + 4$

# Addition

<b>Progression of skills</b>	<b>Key representations</b>		
<b>Bonds within 10</b>  Include bonds for each number within 10  Encourage children to notice patterns.	... is made of ... and ... ... and ... make ...  	... can be partitioned into ... and ...  	... plus ... is equal to ...  $6 + 0 = 6$ $5 + 1 = 6$ $4 + 2 = 6$ $3 + 3 = 6$ $2 + 4 = 6$ $1 + 5 = 6$ $0 + 6 = 6$
<b>Related facts within 20</b>  Make links to known facts.	I know that ... and ... = ... so ... and ... = ...  	... more than ... is ... so ... more than ... is ...  	What patterns do you notice?  $5 + 2 = 7$ $15 + 2 = 17$  $7 = 5 + 2$ $17 = 15 + 2$
<b>Missing numbers</b>  Make links to known facts.	How many more do you need to make ...?  	If ... is the whole and ... is a part, the other part must be...  	... plus ... is equal to ...  $2 + \square = 6$ $6 = 2 + \square$  

## Year 1 Subtraction

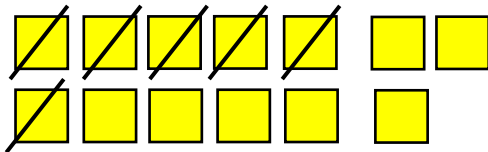
### Concrete

The same range of resources are used as with addition.

### Pictorial

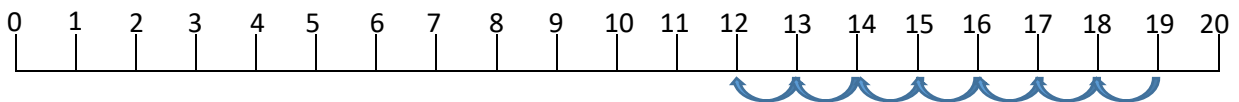
Subtraction by crossing out

$$13 - 6 = \square$$



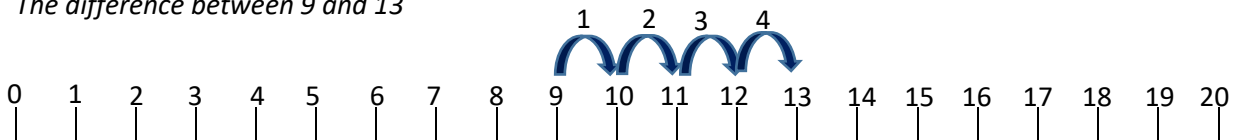
Subtraction by counting back

$$19 - 7 = \square$$



Counting on to find the difference between two numbers:

*The difference between 9 and 13*

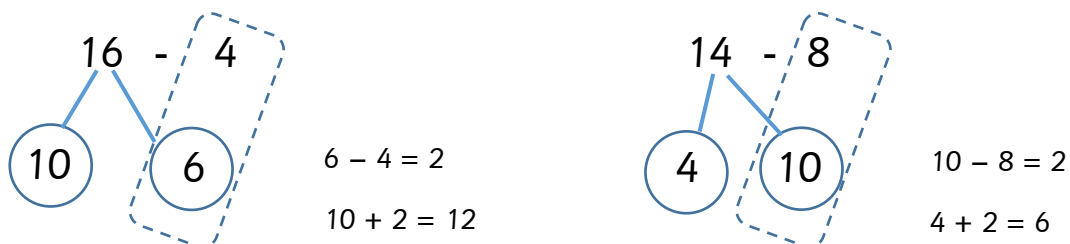


### Abstract

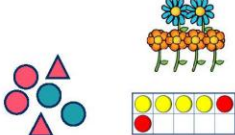
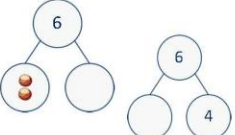
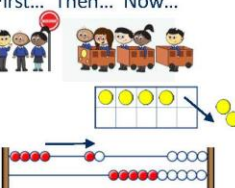
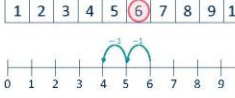
The introduction of subtraction stories.

Start to explore missing number problem involving  $-$  and  $=$  notation.

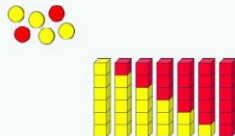
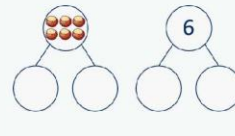
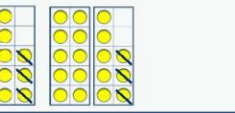
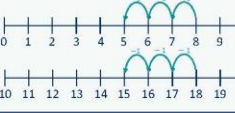
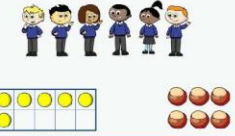
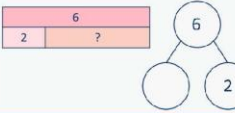
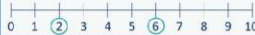
Manipulate numbers with part-part-whole to make calculations easier:



# Subtraction

<b>Year 1</b>	<ul style="list-style-type: none"> <li>Read, write and interpret mathematical statements involving subtraction (−) and equals (=) signs.</li> <li>Represent and use number bonds and related subtraction facts within 20</li> <li>Subtract one-digit and two-digit numbers to 20, including zero.</li> <li>Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = \square - 9</math></li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Find a part</b>  Link to number bonds and known facts. E.g. $2 + 4 = 6$ so if 6 is the whole and 4 is a part, the other part must be 2	There are ... in total. ... are ... How many are <b>not</b> ...?  	... is the whole. ... is a part. ... is a part.  	... subtract ... is equal to ... ... is equal to ... − ...  $6 - 2 = 4$ $6 - 4 = 2$  $4 = 6 - 2$ $2 = 6 - 4$
<b>Take away</b>  A quantity is decreased.	First... Then... Now...  	I start at ... I jump back ... I land on ...  	... minus ... is equal to ... ... is equal to ... − ...  $6 - 2 = 4$ $6 - 4 = 2$  $4 = 6 - 2$ $2 = 6 - 4$

# Subtraction

<b>Progression of skills</b>	<b>Key representations</b>		
<b>Bonds within 10</b>  Focus on subtraction facts.  Encourage children to notice patterns.	... is made of ... and ... ... and ... make ...  	... can be partitioned into ... and ...  	... minus ... is equal to ...  $6 - 0 = 6$ $6 - 1 = 5$ $6 - 2 = 4$ $6 - 3 = 3$ $6 - 4 = 2$ $6 - 5 = 1$ $6 - 6 = 0$
<b>Related facts within 20</b>  Make links to known facts.	I know that ... minus ... = ... so ... minus ... = ...  	... less than ... is ... so ... less than ... is ...  	What patterns do you notice?  $8 - 3 = 5$ $18 - 3 = 15$  $5 = 8 - 3$ $15 = 18 - 3$
<b>Missing numbers</b>  Make links to known facts.	How many do you need to subtract to make ...?  	If ... is the whole and ... is a part, the other part must be...  	... minus ... is equal to ...  $6 - \square = 2$ $2 = 6 - \square$  

## Year 2

### Number – addition and subtraction

#### Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
  - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

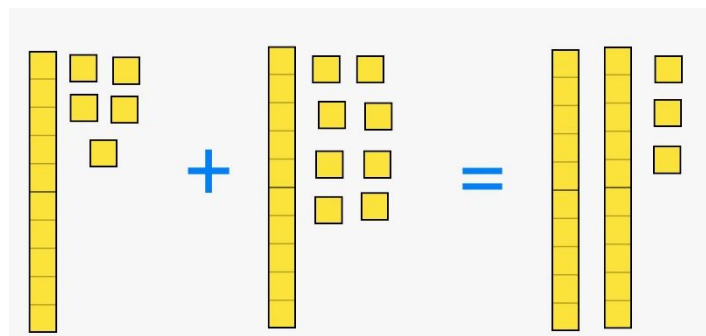
#### Year 2 Addition

##### Concrete

Continue to use resources introduced in Year 1.

In addition to ten frames, card versions are also used.

A greater use is made of Base 10 materials to reinforce the concepts behind the regrouping and renaming when crossing the boundary between ones and tens.

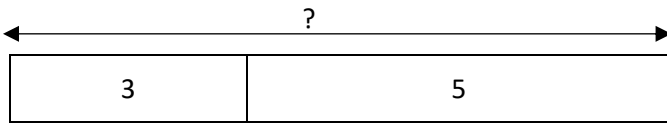


##### Pictorial

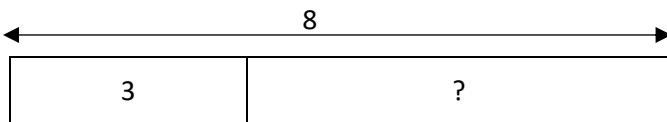
The methods engaged with the Base 10 equipment is 'illustrated' in pupil diagrams.

Year 2 sees an introduction to bar modelling:

$$3 + 5 = \square$$

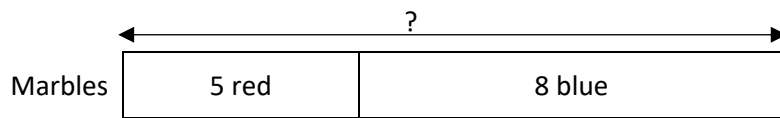


$$3 + \square = 8$$

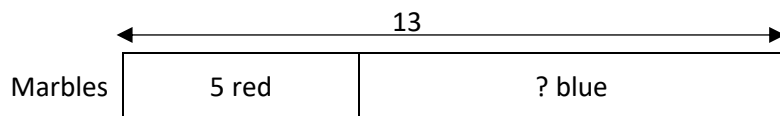


Extending to context based problems such as:

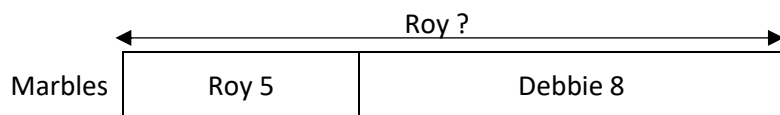
Roy has 5 red marbles and 8 blue marbles. How many marbles does he have altogether? (*Whole unknown*)



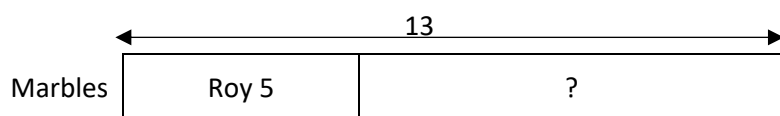
Roy has 13 marbles. Five are red and the rest are blue. How many blue marbles does Roy have? (*Part unknown*)



Roy had 5 marbles. Debbie gave him 8 more marbles. How many marbles does Roy have now? (*Result unknown*)

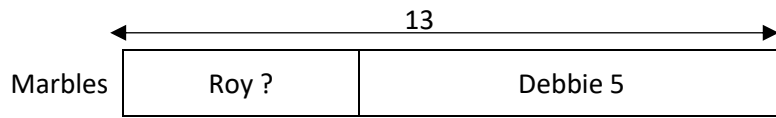


Roy has 5 marbles. How many more marbles does he need to have 13 marbles altogether? (*Change unknown*)





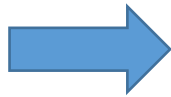
Roy has some marbles. Debbie gave him 5 more marbles. Now he has 13 marbles. How many marbles did Roy have to start with? (*Start unknown*)



**Abstract**

Links between concrete approach and written method are made explicit at each stage.

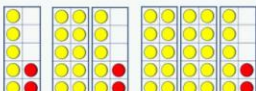

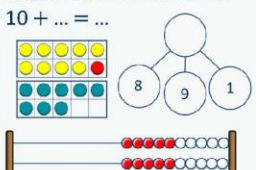
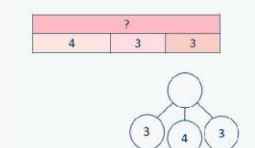
$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 1 \quad 5 \\
 + \quad 1 \quad 8 \\
 \hline
 1 \quad 3 \\
 2 \quad 0 \\
 \hline
 3 \quad 3
 \end{array}$$



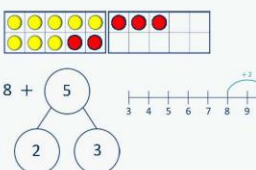
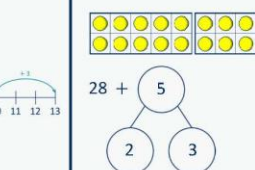
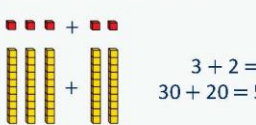
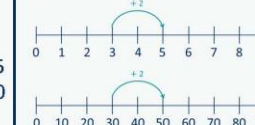
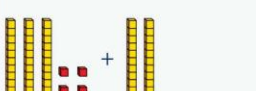

$$\begin{array}{r}
 \text{T} \quad \text{O} \\
 1 \quad 5 \\
 + \quad 1 \quad 8 \\
 \hline
 3 \quad 3 \\
 \hline
 1
 \end{array}$$

Talk through the method, making the links to the concrete method very clear to explain the stages of the calculation.

# Addition

<p><b>Year 2</b></p>	<ul style="list-style-type: none"> <li>Recall and use addition facts to 20 fluently, and derive and use related facts up to 100</li> <li>Add numbers using concrete objects, pictorial representations, and mentally, including:               <ul style="list-style-type: none"> <li>a two-digit number and 1s</li> <li>a two-digit number and 10s</li> <li>2 two-digit numbers</li> <li>adding 3 one-digit numbers</li> </ul> </li> <li>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>		
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Add ones to any number</b> (related facts)</p> <p>Make links to known facts.</p>	<p>I know that ... and ... = ... so ... and ... = ...</p> 	<p>... more than ... is ... so ... more than ... is ...</p> 	<p>What do you notice? Can you continue the pattern?</p> <p><math>5 + 2 = 7</math> <math>15 + 2 = 17</math> <math>25 + 2 = 27...</math></p>
<p><b>Add three 1-digit numbers</b></p> <p>Prompt children to understand that addition can be done in any order and to make links to known facts.</p>	<p>... and ... are a bond to 10 <math>10 + ... = ...</math></p> 	<p>Double ... + ... = ...</p> 	<p>What do you notice? Which addition is the easiest to calculate?</p> <p><math>8 + 9 + 1 =</math> <math>8 + 1 + 9 =</math> <math>9 + 1 + 8 =</math></p>

# Addition

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Add across a 10</b></p> <p>Partition the number being added to make a full ten.</p>	<p>... can be partitioned into ... and ...</p> 	<p>I add ... to get to ... then I add ...</p> <p><math>8 + 5 = 13</math> <math>28 + 5 = 33</math></p> 	
<p><b>Add multiples of 10</b></p> <p>Make links to known facts within ten.</p>	<p>... ones + ... ones = ... ones so ... tens + ... tens = ... tens</p> <p><math>3 + 2 = 5</math> <math>30 + 20 = 50</math></p> 	<p>What is the same? What is different?</p> 	
<p><b>Add 10s to any number</b></p> <p>Make links to known facts.</p>	<p>... tens + ... tens = ... tens ... tens and ... ones = ...</p> 	<p>To add ... I need to add 10 ... times.</p> 	<p>I know that ... and ... = ... so ... and ... = ...</p> <p><math>30 + 20 = 50</math> <math>34 + 20 = 54</math></p>

# Addition

Progression of skills	Key representations																																										
<p><b>Add 2-digit numbers</b> (not across a ten)</p> <p>Lining up ones and tens in columns will support with later written methods.</p>	<p>... ones + ... ones = ... ones ... tens + ... tens = ... tens</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: 1px solid black; padding: 2px;">Tens</td> <td style="border: 1px solid black; padding: 2px;">Ones</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> </table> </div> <div style="text-align: center;"> </div> <div style="text-align: center;"> <table border="1" style="border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">?</td></tr> <tr><td style="width: 20px; height: 20px;">43</td><td style="width: 20px; height: 20px;">21</td></tr> </table> </div> </div> <p>3 ones + 1 one = 4 ones 4 tens + 2 tens = 6 tens 6 tens + 4 ones = 64</p>			Tens	Ones	██████████	██	██████████	██	██████████	██	██████████	██	?	43	21																											
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<p><b>Add 2-digit numbers</b> (across a ten)</p> <p>Begin to exchange 10 ones for 1 ten.</p>	<p>There are .... ones, so I do/do not need to make an exchange.</p> <p>... ones = ... ten and ... ones</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: 1px solid black; padding: 2px;">T</td> <td style="border: 1px solid black; padding: 2px;">O</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██████</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██████</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██████</td> </tr> </table> </div> <div style="font-size: 2em;">→</div> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: 1px solid black; padding: 2px;">T</td> <td style="border: 1px solid black; padding: 2px;">O</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">██████████</td> <td style="border: 1px solid black; padding: 2px;">██</td> </tr> </table> </div> </div> <div style="display: flex; align-items: center; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">?</td></tr> <tr><td style="width: 20px; height: 20px;">45</td><td style="width: 20px; height: 20px;">37</td></tr> </table> </div> <div style="text-align: center;"> </div> </div> <p>5 ones + 7 ones = 12 ones 12 ones = 1 ten and 2 ones 4 tens + 3 tens + 1 ten = 8 tens 8 tens and 2 ones = 82</p>			T	O	██████████	██████	██████████	██████	██████████	██████	T	O	██████████	██	██████████	██	██████████	██	?	45	37																					
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<p><b>Missing numbers</b></p> <p>Solve missing number problems and use the inverse to check.</p>	<p>How many more do you need to make ...?</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td></tr> </table> </div> <div style="text-align: center;"> <math>6 + \square = 10</math>  <math>10 - \square = 6</math> </div> </div> <p>If ... is a whole and ... is a part, then ... is the other part.</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;"> <math>\square + 3 = 7</math>  <math>7 - 3 = \square</math> </div> <div style="text-align: center;"> </div> </div> <p>... can be partitioned into ... and ...</p> <div style="display: flex; align-items: center; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td></tr> <tr><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td></tr> </table> </div> <div style="text-align: center;"> <math>10 + 8 = 12 + \square</math> </div> <div style="border: 1px solid black; padding: 2px;"> <table style="border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td></tr> <tr><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;">●</td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td><td style="width: 20px; height: 20px;"> </td></tr> </table> </div> </div>			●	●	●	●	●	●			●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●						
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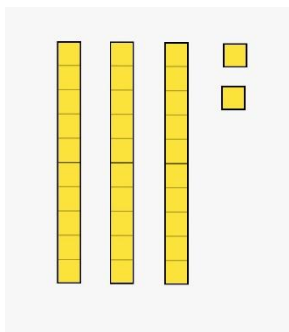
## Year 2 Subtraction

### Concrete

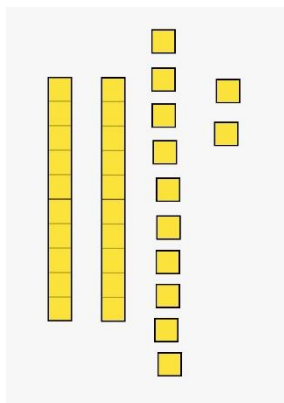
Continue to use resources introduced in Year 1.

Increased use of Base 10 materials to show the stages in a calculation requiring regrouping and renaming.

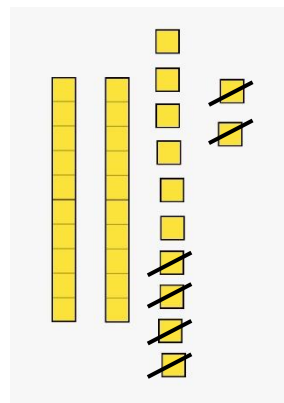
$$32 - 16 = \square$$



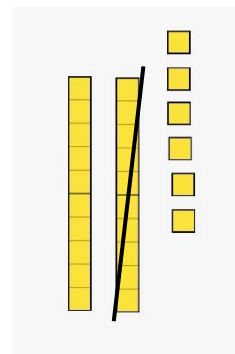
Start with 32



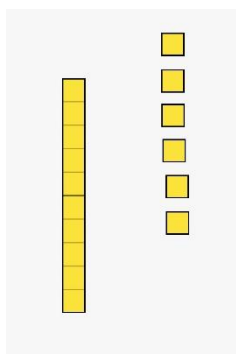
As you cannot subtract 6 ones from 2 ones we need to regroup one of the tens and rename it ten ones.



Now we can subtract 6 ones from the 12 ones.



Then subtract one ten from the two tens.



Leaving the answer 16.

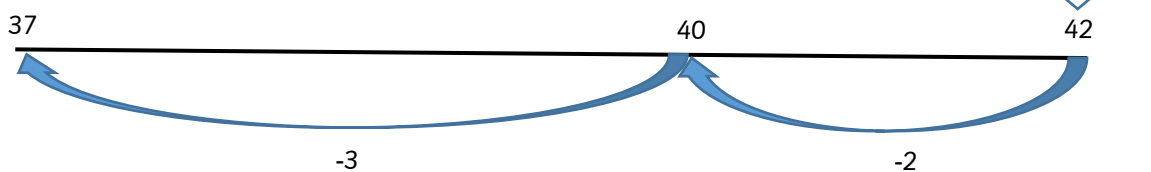
$$32 - 16 = 16$$

### Pictorial

Base 10 methods 'illustrated' in pupil diagrams.

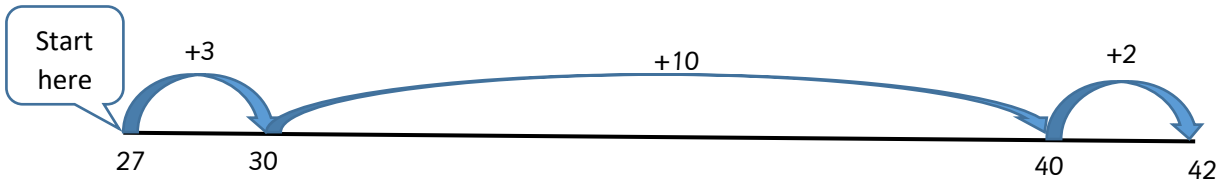
Extend use of number lines with larger "jumps" (including crossing the tens)

$$42 - 5 = \square$$



Use of number lines with 'find the difference' questions.

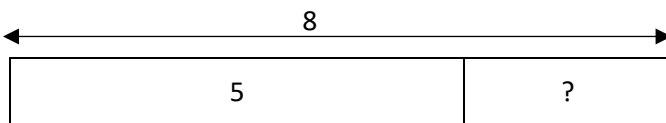
$$42 - 27 = \square$$



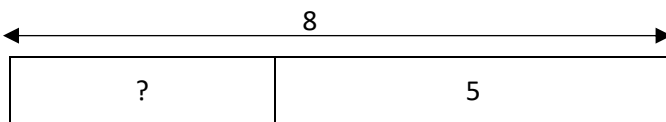
An introduction to bar modelling for subtraction in a variety of formats:

*Part-Part-Whole Problems: Part Unknown*

$$8 - 5 = \square$$

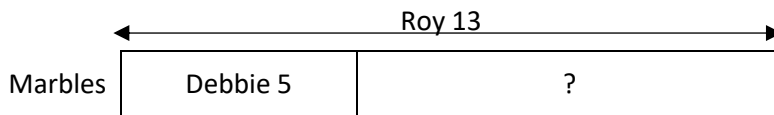


$$8 - \square = 5$$

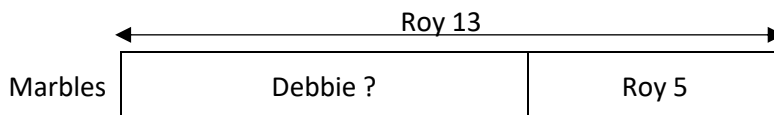


Extending to context based problems such as:

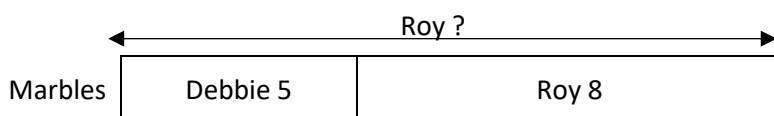
Roy has 13 marbles. He gave 5 to Debbie. How many marbles does Roy have left? (*Result unknown*)



Roy had 13 marbles. He gave some to Debbie. Now he has 5 marbles left. How many marbles did Roy give to Debbie? (*Change unknown*)

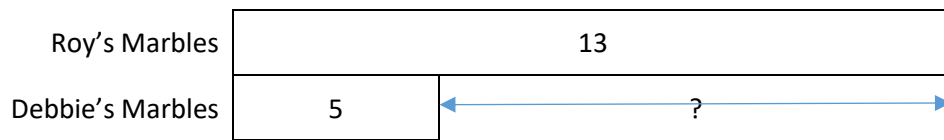


Roy had some marbles. He gave 5 to Debbie. Now he had 8 marbles left. How many marbles did Roy have to start with? (*Start unknown*)

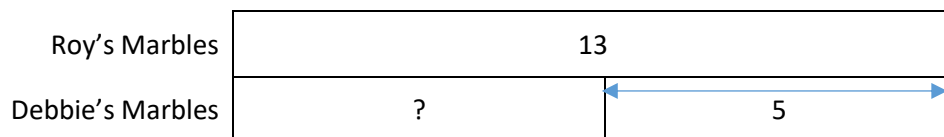


The bar models will also be used for comparing problems:

Roy has 13 marbles. Debbie has 5 marbles. How many more marbles does Roy have than Debbie?  
(Difference unknown)



Roy has 13 marbles. He has 5 more marbles than Debbie. How many marbles does Debbie have?  
(Smallest part unknown)



Debbie has 5 marbles. Roy has 8 more than Debbie. How many marbles does Roy have?  
(Largest part unknown)



**Abstract**

Links between concrete approach and written method are made explicit at each stage.

Formal calculations are initially taught with no need for regrouping and renaming

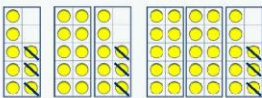
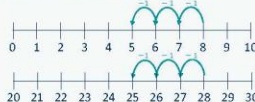
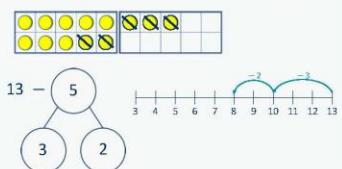
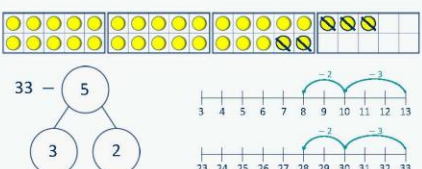
$$\begin{array}{r}
 T \quad O \\
 3 \quad 4 \\
 - 2 \quad 1 \\
 \hline
 1 \quad 3
 \end{array}$$



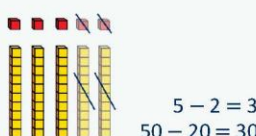
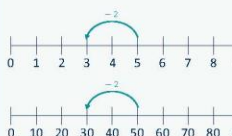

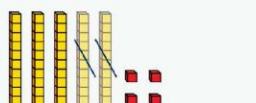
$$\begin{array}{r}
 T \quad O \\
 ~~23~~ \quad 12 \\
 - 1 \quad 6 \\
 \hline
 1 \quad 6
 \end{array}$$

The calculations are "talked through" with the reasoning being made explicit

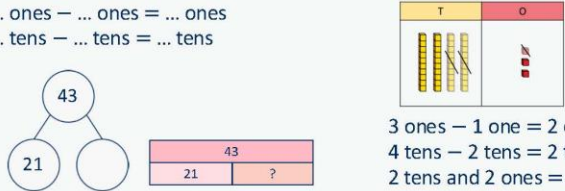
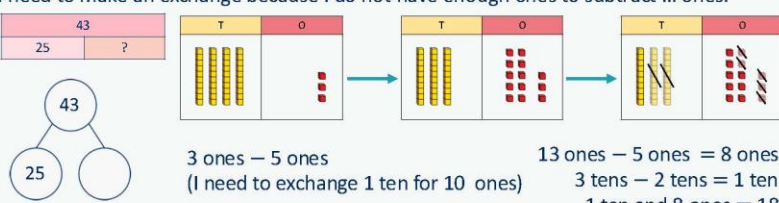
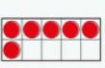
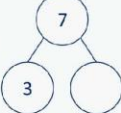
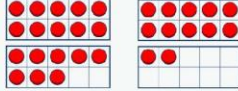
# Subtraction

<p><b>Year 2</b></p>	<ul style="list-style-type: none"> <li>Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100</li> <li>Subtract numbers using concrete objects, pictorial representations, and mentally, including:             <ul style="list-style-type: none"> <li>a two-digit number and 1s</li> <li>a two-digit number and 10s</li> <li>2 two-digit numbers</li> </ul> </li> <li>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>		
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Subtract ones from any number</b> (related facts)</p> <p>Make links to known facts.</p>	<p>I know that ... minus ... = ... so ... minus ... = ...</p> 	<p>... less than ... is ... so ... less than ... is ...</p> 	<p>What do you notice? Can you continue the pattern?</p> <p><math>8 - 3 = 5</math> <math>18 - 3 = 15</math> <math>28 - 3 = 25...</math></p>
<p><b>Subtract across a 10</b></p> <p>Partition the number being subtracted to bridge through a ten.</p>	<p>... can be partitioned into ... and ...</p> 	<p>Make links with related facts.</p> 	

# Subtraction

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>																																																														
<p><b>Subtract multiples of 10</b></p> <p>Make links to known facts within ten.</p>	<p>... ones - ... ones = ... ones so ... tens - ... tens = ... tens</p>  <p><math>5 - 2 = 3</math> <math>50 - 20 = 30</math></p>	<p>What is the same? What is different?</p> 	 <table border="1" data-bbox="1141 1310 1300 1400"> <tr> <td colspan="2">5</td> </tr> <tr> <td>2</td> <td>?</td> </tr> <tr> <td colspan="2">50</td> </tr> <tr> <td>20</td> <td>?</td> </tr> </table>	5		2	?	50		20	?																																																				
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20	?																																																														
<p><b>Subtract 10s from any number</b></p> <p>Make links to known facts.</p>	<p>... tens - ... tens = ... tens ... tens and ... ones = ...</p> 	<p>To subtract ... I need to subtract 10 ... times.</p> <table border="1" data-bbox="821 1512 1053 1646"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	<p>I know that ... minus ... = ... so ... minus ... = ...</p> <p><math>50 - 20 = 30</math> <math>54 - 20 = 34</math></p>
1	2	3	4	5	6	7	8	9	10																																																						
11	12	13	14	15	16	17	18	19	20																																																						
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41	42	43	44	45	46	47	48	49	50																																																						
51	52	53	54	55	56	57	58	59	60																																																						

# Subtraction

Progression of skills	Key representations		
<p><b>Subtract two 2-digit numbers</b> (not across a ten)</p>	<p>... ones – ... ones = ... ones ... tens – ... tens = ... tens</p>  <p>3 ones – 1 one = 2 ones 4 tens – 2 tens = 2 tens 2 tens and 2 ones = 22</p>		
<p><b>Subtract two 2-digit numbers</b> (across a ten)</p> <p>Begin to exchange 1 ten for 10 ones.</p>	<p>I need to make an exchange because I do not have enough ones to subtract ... ones.</p>  <p>3 ones – 5 ones (I need to exchange 1 ten for 10 ones)</p> <p>13 ones – 5 ones = 8 ones 3 tens – 2 tens = 1 ten 1 ten and 8 ones = 18</p>		
<p><b>Missing numbers</b></p> <p>Solve missing number problems and use the inverse to check.</p>	<p>How many do you need to subtract to make ...?</p>  <p><math>10 - \square = 6</math> <math>6 + \square = 10</math></p>	<p>If ... is a whole and ... is a part, then ... is the other part.</p> <p><math>7 - 3 = \square</math> <math>\square + 3 = 7</math></p> 	<p>... can be partitioned into ... and ...</p> <p><math>18 - \square = 12 + 2</math></p> 



## Year 3

### Number – addition and subtraction

#### Statutory requirements

Pupils should be taught to:

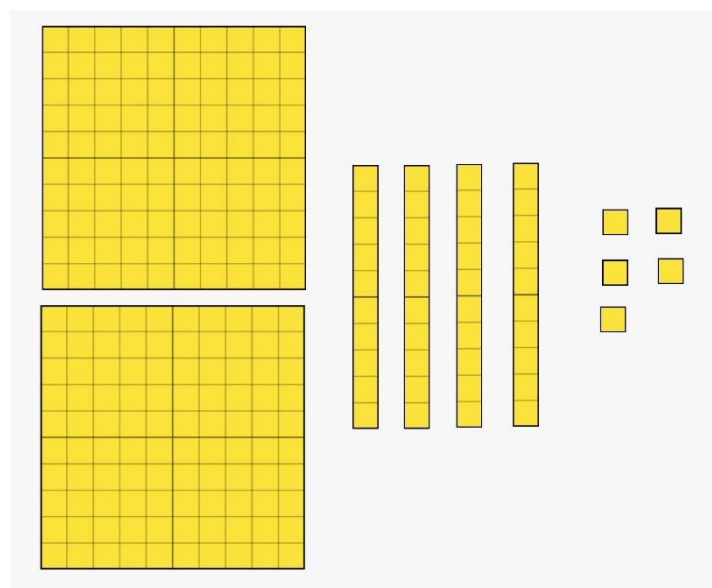
- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

### Year 3 Addition

#### Concrete

As new concepts are taught, reference continues to be made to the concrete and visual representations from earlier in the school to reinforce the reasoning behind the calculations.

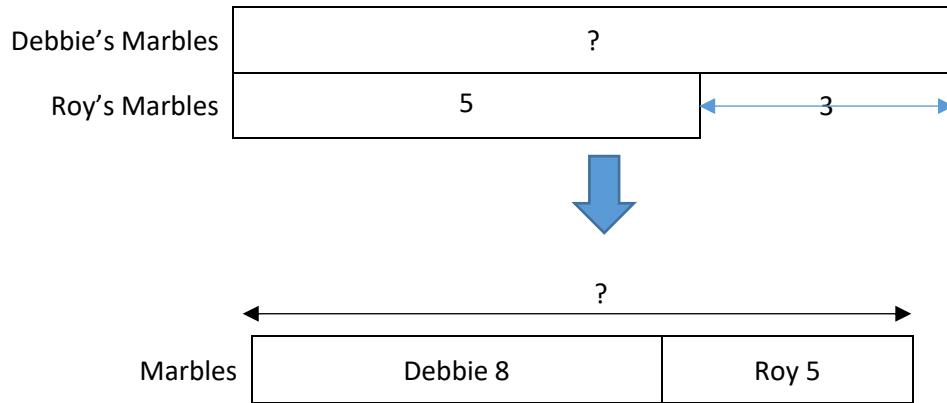
Base 10 materials continue to be used heavily to represent number.



### Pictorial

The mastery of additive reason problems from KS1 is checked, with revision as necessary. Develop use of bar model with two step joining problems:

Roy has 5 marbles. Debbie has 3 more than Roy. How many marbles do they both have altogether?



### Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation.

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 2 \quad 7 \quad 8 \\ + \quad \quad 8 \quad 2 \\ \hline \quad 1 \quad 0 \\ 1 \quad 5 \quad 0 \\ 2 \quad 0 \quad 0 \\ \hline 3 \quad 6 \quad 0 \end{array}$$



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 2 \quad 7 \quad 8 \\ + \quad \quad 8 \quad 2 \\ \hline 3 \quad 6 \quad 0 \end{array}$$

~~1~~ ~~1~~

Cross off the number after it has been added to the other numbers in that column




# Addition

<b>Year 3</b>	<ul style="list-style-type: none"> <li>Add numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.</li> <li>Add numbers with up to three digits, using formal written methods of columnar addition.</li> <li>Add fractions with the same denominator within 1 whole.</li> <li>Calculate the time taken by particular events or tasks.</li> </ul>																									
<b>Progression of skills</b>	<b>Key representations</b>																									
<p><b>Add 1s, 10s or 100s to a 3-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds column will increase by ...</p> <table border="1"> <tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr> <tr><td>400</td><td>40</td><td>4</td></tr> </table> <p>444 + 5 =                      777 + 2 =</p> <p>444 + 50 =                    777 + 20 =</p> <p>444 + 500 =                  777 + 200 =</p>	Hundreds	Tens	Ones	400	40	4	<p>What patterns do you notice?</p> <p>235 + 3 =</p> <p>235 + 30 =</p> <p>235 + 300 =</p> <p>604 + 20 =                    111 + <input type="text"/> = 118</p> <p>604 + 50 =                    111 + <input type="text"/> = 181</p> <p>604 + 90 =                    111 + <input type="text"/> = 811</p>																		
Hundreds	Tens	Ones																								
400	40	4																								
<p><b>Add two numbers (no exchange)</b></p> <p>Mental strategies and introduction of formal written method.</p>	<p>... ones + ... ones = ... ones</p> <p>... tens + ... tens = ... tens</p> <p>... hundreds + ... hundreds = ... hundreds</p> <p>345      432</p> <table border="1"> <tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr> <tr><td>300</td><td>40</td><td>5</td></tr> <tr><td>400</td><td>30</td><td>2</td></tr> </table>	Hundreds	Tens	Ones	300	40	5	400	30	2	<table border="1"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>3</td><td>4</td><td>5</td></tr> <tr><td>+</td><td>4</td><td>3</td></tr> <tr><td colspan="3">-----</td></tr> <tr><td></td><td></td><td></td></tr> </table>	H	T	O	3	4	5	+	4	3	-----					
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# Addition

<b>Progression of skills</b>	<b>Key representations</b>																																																							
<p><b>Add two numbers across a 10 or 100</b></p> <p>Formal written method involving up to 2 exchanges including 3-digit plus 2-digit numbers.</p>	<p>There are ... ones, so I do/do not need to make an exchange.</p> <p>There are ... tens, so I do/do not need to make an exchange.</p> <p>... ones = ... ten and ... ones.</p> <p>... tens = ... hundred and ... tens.</p> <p>255      54</p> <table border="1"> <tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr> <tr><td>200</td><td>50</td><td>5</td></tr> <tr><td></td><td>5</td><td>4</td></tr> </table> <p>466      353</p> <table border="1"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>4</td><td>6</td><td>6</td></tr> <tr><td>+</td><td>3</td><td>5</td></tr> <tr><td colspan="3">-----</td></tr> <tr><td></td><td></td><td></td></tr> </table>	Hundreds	Tens	Ones	200	50	5		5	4	H	T	O	4	6	6	+	3	5	-----						<table border="1"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>2</td><td>5</td><td>5</td></tr> <tr><td>+</td><td>5</td><td>4</td></tr> <tr><td colspan="3">-----</td></tr> <tr><td></td><td></td><td></td></tr> </table> <table border="1"> <tr><th>H</th><th>T</th><th>O</th></tr> <tr><td>3</td><td>6</td><td>6</td></tr> <tr><td>+</td><td>1</td><td>6</td></tr> <tr><td colspan="3">-----</td></tr> <tr><td></td><td></td><td></td></tr> </table>	H	T	O	2	5	5	+	5	4	-----						H	T	O	3	6	6	+	1	6	-----					
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<p><b>Complements to 100</b></p> <p>Pairs of numbers which total 100</p>	<p>... plus ... is equal to 100</p> <p>38      ?</p> <table border="1"> <tr><td>100</td></tr> <tr><td>38      ?</td></tr> </table>	100	38      ?	<p>I add ... to get to the next 10, then ... to get to 100</p> <p>38 + 62 = 100</p> <p>62 + 38 = 100</p> <p>100 = 38 + 62</p> <p>100 = 62 + 38</p> <p>38      40      100</p> <p>+2                      +60</p>																																																				
100																																																								
38      ?																																																								

# Addition

Progression of skills	Key representations
<p><b>Add fractions with the same denominator within 1 whole</b></p> <p>Make links with known facts.</p>	<p>When adding fractions with the same denominator, I only add the numerator. ... fifths + ... fifths = ... fifths</p> <div style="display: flex; align-items: center; gap: 10px;"> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #008080; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #ffff00; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0;"></div> </div> $\frac{1}{5} + \frac{1}{5}$ <div style="display: flex; align-items: center; gap: 10px;"> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #008080; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #ffff00; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #ffff00; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0;"></div> </div> $\frac{1}{5} + \frac{2}{5}$ <div style="display: flex; align-items: center; gap: 10px;"> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #008080; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #ffff00; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #ffff00; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0; margin-right: 5px;"></div> <div style="border: 1px solid black; width: 40px; height: 15px; background-color: #e0e0e0;"></div> </div> $\frac{1}{5} + \frac{3}{5}$
<p><b>Calculate the duration of events</b></p> <p>Find durations of time between a given start and end point. Children will need to calculate complements to 60</p>	<p>From ... to ... o'clock is ... minutes. From ... o'clock to ... is ... minutes. The total time taken is ... minutes.</p> <div style="display: flex; align-items: center; gap: 20px;"> <div style="text-align: center;">  <p>start</p> </div> <div style="text-align: center;">  <p>finish</p> </div> <div style="text-align: center;"> <div style="border: 1px solid black; padding: 2px 5px; display: inline-block;">4:25</div> <div style="border: 1px solid black; padding: 2px 5px; display: inline-block; margin-left: 10px;">4:55</div> <p>start      finish</p> </div> <div style="text-align: center;">  <p>2:25      3:00      3:18</p> </div> </div>

## Year 3 Subtraction

### Concrete

As new concepts are taught, reference continues to be made to the concrete and visual representations from earlier in the school to reinforce the reasoning behind the calculations.

Base 10 materials continue to be used heavily to represent numbers in the calculations.

### Pictorial

Full understanding of bar modelling from KS1 is checked and revised as necessary.

### Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation, beginning with three digit columnar written strategies. Initially with no exchange, then with the regrouping of tens into ones (as initially introduced in Year 2).

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 5 \quad \cancel{2} \quad ^1 2 \\ - 3 \quad 1 \quad 6 \\ \hline 2 \quad 1 \quad 6 \end{array}$$




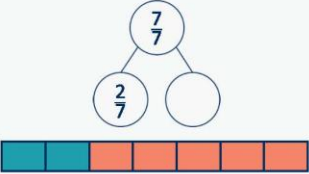
# Subtraction

<b>Year 3</b>	<ul style="list-style-type: none"> <li>Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.</li> <li>Subtract numbers with up to three digits, using formal written methods.</li> <li>Subtract fractions with the same denominator within 1 whole.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<p><b>Subtract 1s, 10s and 100s from a 3-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds column will decrease by ...</p> <p> <math>444 - 2 =</math>                      <math>777 - 4 =</math>  <math>444 - 20 =</math>                    <math>777 - 40 =</math>  <math>444 - 200 =</math>                  <math>777 - 400 =</math> </p>	<p>What patterns do you notice?</p> <p> <math>235 - 3 =</math>  <math>235 - 30 =</math>  <math>235 - 300 =</math> </p> <p> <math>118 - \square = 111</math>  <math>624 - 20 =</math>    <math>181 - \square = 111</math>  <math>654 - 50 =</math>    <math>811 - \square = 111</math>  <math>694 - 90 =</math> </p>
<p><b>Subtract two numbers (no exchange)</b></p> <p>Mental strategies and introduction of formal written method.</p>	<p>... ones - ... ones = ... ones          ... tens - ... tens = ... tens          ... hundreds - ... hundreds = ... hundreds</p>	

# Subtraction

<b>Progression of skills</b>	<b>Key representations</b>	
<p><b>Subtract two numbers across a 10 or 100</b></p> <p>Formal written method involving up to 2 exchanges including 3-digit subtract 2-digit numbers.</p>	<p>I need to subtract ... ones. I do/do not need to make an exchange.          I need to subtract ... tens. I do/do not need to make an exchange.          I can exchange 1 ... for 10 ...</p>	
<p><b>Complements to 100</b></p> <p>Focus on subtraction facts. Encourage children to notice patterns.</p>	<p>100 minus ... is equal to ...</p>	<p>I subtract ... tens, then I subtract ... ones.</p> <p> <math>100 - 38 = 62</math>  <math>100 - 62 = 38</math>  <math>62 = 100 - 38</math>  <math>38 = 100 - 62</math> </p>

# Subtraction

Progression of skills	Key representations
<p><b>Subtract fractions with the same denominator within 1 whole</b></p> <p>Make links with known facts.</p>	<p>When subtracting fractions with the same denominator, I only subtract the numerator. ... fifths – ... fifths = ... fifths</p> <div style="display: flex; align-items: center; margin-bottom: 5px;">  <math>\frac{5}{5} - \frac{1}{5}</math> </div> <div style="display: flex; align-items: center; margin-bottom: 5px;">  <math>\frac{4}{5} - \frac{1}{5}</math> </div> <div style="display: flex; align-items: center;">  <math>\frac{3}{5} - \frac{1}{5}</math> </div> <div style="text-align: right; margin-top: 20px;">  </div>

# Year 4

## Number – addition and subtraction

### Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

### Year 4 Addition

#### Abstract

Continue to model concrete and visual representations practically alongside the formal written calculation moving into 4 digit calculations.

$$\begin{array}{rcccc} & \text{Th} & \text{H} & \text{T} & \text{O} \\ & 7 & 3 & 1 & 0 \\ + & 5 & 9 & 3 & 8 \\ \hline 1 & 3 & 2 & 4 & 8 \\ \hline \cancel{\phantom{1}} & \cancel{\phantom{3}} & & & \end{array}$$

Begin to explore decimals in the context of money




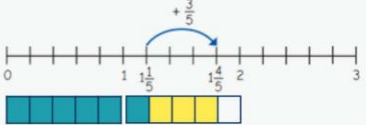
$$\begin{array}{rcccc} & \text{T} & \text{O} & \cdot & \frac{1}{10} & \frac{1}{100} \\ \pounds & 1 & 8 & \cdot & 7 & 2 \\ + & \pounds & 5 & 9 & \cdot & 2 & 9 \\ \hline \pounds & 7 & 8 & \cdot & 0 & 1 \\ \hline \cancel{\phantom{\pounds}} & \cancel{\phantom{7}} & \cancel{\phantom{8}} & & \cancel{\phantom{0}} & \end{array}$$



# Addition

<p><b>Year 4</b></p>	<ul style="list-style-type: none"> <li>Add numbers with up to 4 digits using a formal written method.</li> <li>Solve simple measure and money problems involving fractions and decimals to 2 decimal places.</li> <li>Add fractions with the same denominator.</li> </ul>																																	
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>																																	
<p><b>Add 1s, 10s and 100s to a 4-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds/thousands column will increase by ...</p> <table border="1" data-bbox="531 546 799 656"> <thead> <tr> <th>Thousands</th> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>4</td> <td>2</td> <td>5</td> </tr> </tbody> </table> <p>3,425 + 3 =            3,425 + 300 = 3,425 + 30 =        3,425 + 3,000 =</p>	Thousands	Hundreds	Tens	Ones	3	4	2	5	<p>What patterns do you notice?</p> <p>2,350 + 3 = 2,350 + 30 = 2,350 + 300 = 2,350 + 3,000 =</p> <p>6,040 + 200 =    2,211 + <input type="text"/> = 2,251 6,040 + 500 =    2,211 + <input type="text"/> = 2,215 6,040 + 900 =    2,211 + <input type="text"/> = 2,511</p>																								
Thousands	Hundreds	Tens	Ones																															
3	4	2	5																															
<p><b>Add up to two 4-digit numbers</b></p> <p>Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.</p>	<p>There are ... ones/tens/hundreds so I do/do not need to make an exchange.</p> <p>I can exchange 10 ... for 1 ...</p> <table border="1" data-bbox="906 752 1088 965"> <thead> <tr> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>4</td> <td>2</td> <td>5</td> </tr> <tr> <td>3</td> <td>4</td> <td>2</td> <td>5</td> </tr> </tbody> </table> <table border="1" data-bbox="1137 792 1286 920"> <thead> <tr> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td>4</td> <td>6</td> <td>7</td> <td>3</td> </tr> <tr> <td>+</td> <td>1</td> <td>5</td> <td>1</td> </tr> <tr> <td colspan="4">-----</td> </tr> <tr> <td>6</td> <td>1</td> <td>9</td> <td>1</td> </tr> </tbody> </table>		Th	H	T	O	3	4	2	5	3	4	2	5	Th	H	T	O	4	6	7	3	+	1	5	1	-----				6	1	9	1
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# Addition

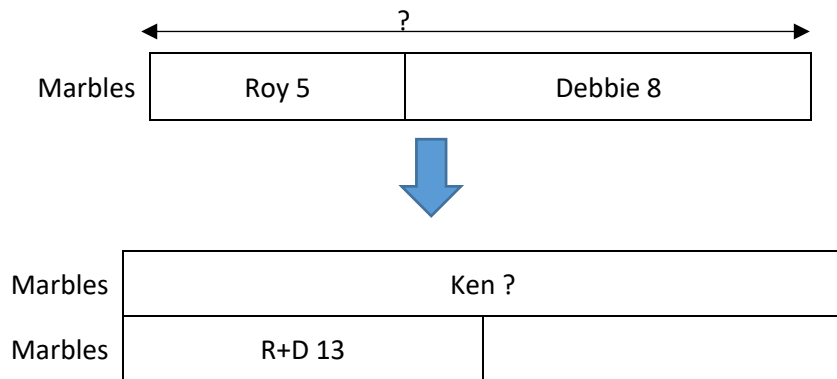
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Add decimal numbers in the context of money</b></p> <p>Emphasis on partitioning and use of number lines rather than formal written calculations.</p>	<p>... pence + ... pence = ... pence ... pounds + ... pounds = ... pounds</p>  <p>45p + 25p = 70p £2 + £3 = £5 £5 + 70p = £5.70</p>	<p>£3.25 can be partitioned into £3 + 20p + 5p</p> 
<p><b>Add fractions and mixed numbers with the same denominator beyond 1 whole</b></p>	<p>When adding fractions with the same denominator, I only add the numerator. ... fifths + ... fifths = ... fifths</p> <p><math>\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}</math></p>  	

## Year 4 Subtraction

### Pictorial

Develop repertoire of bar modelling to include 2 step separating problems:

Roy has 5 marbles and Debbie has 8 marbles. Ken says, "I have double the number of marbles you have together." How many marbles does Ken have?



### Abstract

As before, continue to model and reinforce with concrete resources and visual representations throughout.

Extend to subtraction of 4 digit numbers with exchanging (regrouping and renaming) now introduced from hundreds into tens, building to thousands into hundreds. Initially, questions will be asked where only one exchange is needed.

$$\begin{array}{r}
 \text{Th} \quad \text{H} \quad \text{T} \quad \text{O} \\
 5 \quad \cancel{2} \quad 13 \quad 9 \\
 - 3 \quad 1 \quad 4 \quad 6 \\
 \hline
 2 \quad 1 \quad 9 \quad 3
 \end{array}$$

Begin to explore decimals in the context of money

$$\begin{array}{r}
 \text{T} \quad \text{O} \cdot \frac{1}{10} \quad \frac{1}{100} \\
 \pounds \quad 9 \quad \cancel{0} \cdot 18 \quad 5 \\
 - \pounds \quad 5 \quad 0 \cdot 9 \quad 1 \\
 \hline
 \pounds \quad 4 \quad 0 \cdot 9 \quad 4
 \end{array}$$

# Subtraction

<b>Year 4</b>	<ul style="list-style-type: none"> <li>Subtract numbers with up to 4 digits using a formal written method.</li> <li>Solve simple measure and money problems involving fractions and decimals to 2 decimal places.</li> <li>Subtract fractions with the same denominator.</li> </ul>																																	
<b>Progression of skills</b>	<b>Key representations</b>																																	
<p><b>Subtract 1s, 10s, 100s and 1,000s from a 4-digit number</b></p> <p>Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.</p>	<p>The ones/tens/hundreds/thousands column will decrease by ...</p> <table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th style="background-color: #d9ead3;">Thousands</th> <th style="background-color: #d9ead3;">Hundreds</th> <th style="background-color: #d9ead3;">Tens</th> <th style="background-color: #d9ead3;">Ones</th> </tr> </thead> <tbody> <tr> <td>400 300 100</td> <td>300 200 100</td> <td>20 10</td> <td>5 4</td> </tr> </tbody> </table> <p> <math>3,425 - 2 = 3,425 - 200 =</math>  <math>3,425 - 20 = 3,425 - 2,000 =</math> </p>	Thousands	Hundreds	Tens	Ones	400 300 100	300 200 100	20 10	5 4	<p>What patterns do you notice?</p> <p> <math>4,356 - 3 =</math>  <math>4,356 - 30 =</math>  <math>4,356 - 300 =</math>  <math>4,356 - 3,000 =</math> </p> <p> <math>4,433 - \square = 4,430</math>  <math>4,433 - \square = 4,033</math>  <math>4,433 - \square = 4,403</math> </p>																								
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400 300 100	300 200 100	20 10	5 4																															
<p><b>Subtract up to two 4-digit numbers</b></p> <p>Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.</p>	<p>I need to subtract... ones/tens/hundreds. I do/do not need to make an exchange.</p> <p>I can exchange 1... for 10...</p> <div style="display: flex; align-items: center;"> <table border="1" style="margin-right: 20px;"> <thead> <tr> <th style="background-color: #d9ead3;">Th</th> <th style="background-color: #d9ead3;">H</th> <th style="background-color: #d9ead3;">T</th> <th style="background-color: #d9ead3;">O</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>4</td> <td>2</td> <td>5</td> </tr> <tr> <td>2</td> <td>1</td> <td>4</td> <td>8</td> </tr> <tr> <td>1</td> <td>0</td> <td>5</td> <td>8</td> </tr> </tbody> </table> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th style="background-color: #d9ead3;">Th</th> <th style="background-color: #d9ead3;">H</th> <th style="background-color: #d9ead3;">T</th> <th style="background-color: #d9ead3;">O</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>4</td> <td>2</td> <td>5</td> </tr> <tr> <td>-</td> <td>2</td> <td>1</td> <td>4</td> </tr> <tr> <td>1</td> <td>0</td> <td>5</td> <td>8</td> </tr> </tbody> </table> </div>		Th	H	T	O	3	4	2	5	2	1	4	8	1	0	5	8	Th	H	T	O	3	4	2	5	-	2	1	4	1	0	5	8
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# Subtraction

<b>Progression of skills</b>	<b>Key representations</b>	
<p><b>Subtract decimal numbers in the context of money</b></p> <p>Emphasis here is on partitioning and use of number lines rather than formal written calculations.</p>	<p>I can partition £... into £... and 100p</p> <p>£... - £... = £... 100p - ...p = ...p</p> <p>£5 - £3.26 £4 - £3 = £1 100p - 26p = 74p £5 - £3.26 = £1.74</p> <div style="text-align: center;"> </div>	<p>£3.26 can be partitioned into £3 + 20p + 6p</p> <div style="text-align: center;"> </div>
<p><b>Subtract fractions and mixed numbers with the same denominator</b></p> <p>Include subtracting fractions from wholes.</p>	<p>When subtracting fractions with the same denominator, I only subtract the numerator. ... tenths - ... tenths = ... tenths</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <p><math>\frac{16}{10} - \frac{5}{10}</math></p> </div> <div style="margin-right: 20px;"> <p><math>\frac{16}{10} - \frac{9}{10}</math></p> </div> <div> </div> </div>	


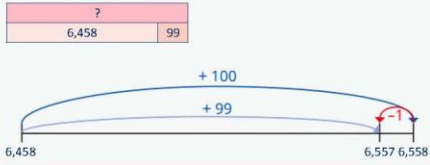
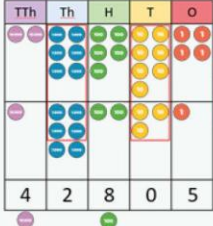
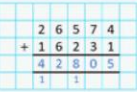
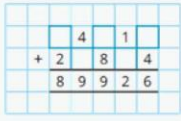


Progress to addition of numbers to two decimal places in context (such as money or measurement).


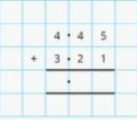
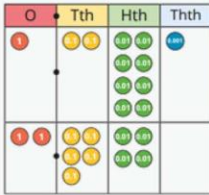

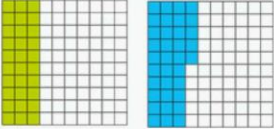
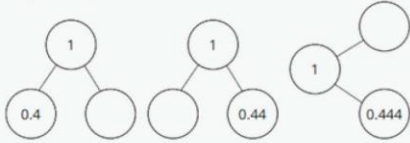
$$\begin{array}{rcccccc} & & \text{H} & \text{T} & \text{O} & \cdot & \frac{1}{10} & \frac{1}{100} \\ & \text{£} & 5 & 1 & 8 & \cdot & 7 & 2 \\ + & \text{£} & 3 & 5 & 9 & \cdot & 2 & 9 \\ \hline & \text{£} & 8 & 7 & 8 & \cdot & 0 & 1 \\ \hline & & & & & & \cancel{7} & \cancel{2} & \cancel{9} \end{array}$$

As throughout, consider the appropriateness of the numbers, initially starting with one carry to ensure clarity and understanding of the layout and process before gradually increasing the complexity of the calculations.

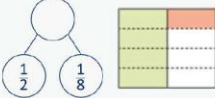
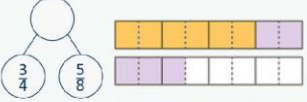
# Addition

<b>Year 5</b>	<ul style="list-style-type: none"> <li>Add whole numbers with more than 4 digits, including using formal written methods.</li> <li>Add numbers mentally with increasingly large numbers.</li> <li>Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1</li> <li>Add fractions with the same denominator, and denominators that are multiples of the same number.</li> </ul>
<b>Progression of skills</b>	<b>Key representations</b>
<b>Add using mental strategies</b>  Add 1s, 10s, 100s, etc. to any number. Use number bonds and related facts.	 $48,650 + 300 =$ $48,650 + 30,000 =$ $48,650 + 30 =$
<b>Add whole numbers with more than 4 digits</b>  Encourage children to estimate and use inverse operations to check answers to calculations.	To add ..., I can add ... then subtract ...    

# Addition

<b>Progression of skills</b>	<b>Key representations</b>
<b>Add decimals with up to 2 decimal places</b>  Progress from the same number of decimal places to a different number of decimal places, and from no exchange to exchange.	I do/do not need to make an exchange because ... I can exchange 10 ... for 1 ...    
<b>Complements to 1</b>  Pairs of numbers with up to 3 decimal places which total 1  Encourage children to make links with bonds to 10 and complements to 100 and 1,000	 $0.3 + \square = 1$ $0.35 + \square = 1$  $4 + 6 = 10$ $0.4 + 0.6 = 1$ $44 + 56 = 100$ $0.44 + 0.56 = 1$ $444 + 556 = 1,000$ $0.444 + 0.556 = 1$

# Addition

Progression of skills	Key representations
<p><b>Add fractions with denominators that are a multiple of one another</b></p> <p>Encourage children to convert fractions to the same denominator before adding.</p> <p>Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.</p>	<p>The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <math display="block">\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}</math> </div> <div style="text-align: center;">  <math display="block">\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}</math> </div> </div>

## Year 5 Subtraction

### Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent.

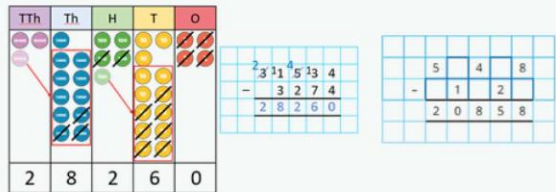
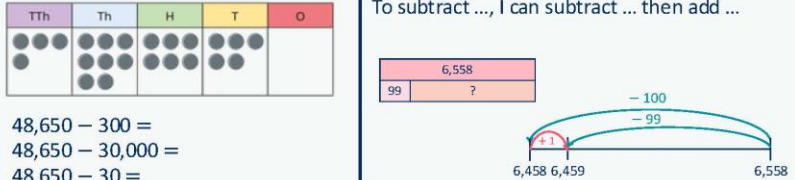
Strategies build on those of Year 4 and involve starting with numbers up to 100,000 and progressing to 1,000,000.

Progressively, and before moving to larger numbers, begin to explore written strategies where '2 exchanges' are needed.

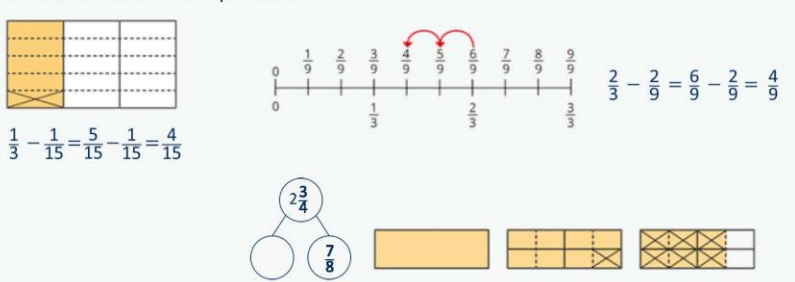
	Th	H	T	O
	7	<del>8</del> <sup>9</sup>	<del>10</del> <sup>9</sup>	16
-	2	5	9	8
<hr/>				
	5	3	0	8
<hr/>				



# Subtraction

<p><b>Year 5</b></p>	<ul style="list-style-type: none"> <li>Subtract whole numbers with more than 4 digits.</li> <li>Subtract numbers mentally with increasingly large numbers.</li> <li>Subtract decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1</li> <li>Subtract fractions with the same denominator, and denominators that are multiples of the same number.</li> </ul>
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Subtract whole numbers with more than 4 digits</b></p> <p>Encourage children to estimate and use inverse operations to check answers to calculations.</p>	<p>I can exchange 1 ... for 10 ...</p> 
<p><b>Subtract using mental strategies</b></p> <p>Subtract 1s, 10s, 100s etc from any number. Use number bonds and related facts.</p>	<p>To subtract ..., I can subtract ... then add ...</p>  <p> <math>48,650 - 300 =</math>  <math>48,650 - 30,000 =</math>  <math>48,650 - 30 =</math> </p>

# Subtraction

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Subtract fractions with denominators that are a multiple of one another</b></p> <p>Convert fractions to the same denominator before subtracting. Progress from subtracting fractions within 1 whole to subtracting from a mixed number.</p>	<p>The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.</p>  <p> <math>\frac{1}{3} - \frac{1}{15} = \frac{5}{15} - \frac{1}{15} = \frac{4}{15}</math> </p> <p> <math>\frac{2}{3} - \frac{2}{9} = \frac{6}{9} - \frac{2}{9} = \frac{4}{9}</math> </p>

## Year 6

### Number – addition and subtraction

#### Statutory requirements

Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

#### Year 6 Addition

##### Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent. Strategies build on those of Year 4 and Year 5 with numbers within 10 million and calculations with up to 3 decimal places (through problems set in contextual situations, such as measurement).

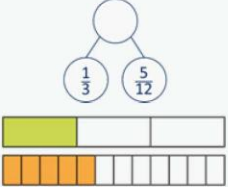
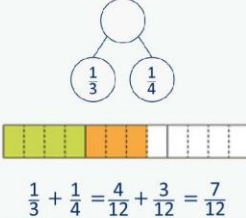
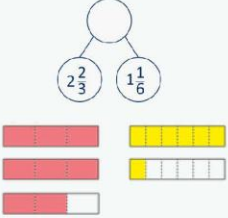
# Addition

<p><b>Year 6</b></p>	<ul style="list-style-type: none"> <li>• Add larger numbers, using the formal written method of columnar addition.</li> <li>• Use their knowledge of the order of operations to carry out calculations involving the 4 operations.</li> <li>• Calculate intervals across zero.</li> <li>• Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</li> </ul>
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Add integers up to 10 million</b></p> <p>Encourage children to estimate and use inverse operations to check answers to calculations.</p>	
<p><b>Add decimals with up to 3 decimal places</b></p> <p>Progress to numbers with digits in different place value columns.</p> <p>Encourage children to check that they have lined up the columns correctly.</p>	<p>I do/do not need to make an exchange because ...</p>

# Addition

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Order of operations</b></p> <p>Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction. *When no brackets are shown and the operations have the same priority, work left to right.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p>	
<p><b>Negative numbers</b></p> <p>Children add to negative numbers and carry out calculations which cross 0</p>	<p>... plus ... is equal to ...</p>	<p>The difference between <math>-5</math> and <math>-1</math> is 4</p> <p>The difference between <math>-5</math> and 5 is 10</p>

# Addition

Progression of skills	Key representations		
<p><b>Add fractions</b></p> <p>Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.</p>	<p>The denominator has been multiplied by ..., so the numerator needs to be multiplied by ...</p> 	<p>The lowest common multiple of ... and ... is ...</p>  $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$	<p>...is made up of ... wholes and ...</p> 

## Year 6 Subtraction

### Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent. Strategies build on those of Year 4 and Year 5 with numbers within 10 million and calculations with up to 3 decimal places (through problems set in contextual situations, such as measurement).

$$\begin{array}{r} \overset{2}{\cancel{3}} \quad 15 \cdot \overset{6}{\cancel{7}} \quad \overset{10}{\cancel{1}} \quad 12 \\ - \quad \quad 8 \cdot 6 \quad 5 \quad 3 \\ \hline 2 \quad 7 \cdot 0 \quad 5 \quad 9 \\ \hline \end{array}$$

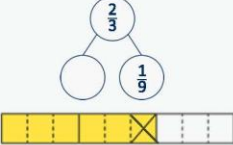
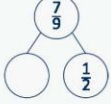
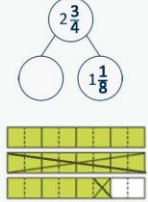
# Subtraction

<b>Year 6</b>	<ul style="list-style-type: none"> <li>Subtract larger numbers, using the formal written methods of columnar subtraction.</li> <li>Use their knowledge of the order of operations to carry out calculations involving the 4 operations.</li> <li>Calculate intervals across zero.</li> <li>Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.</li> </ul>
<b>Progression of skills</b>	<b>Key representations</b>
<p><b>Subtract integers up to 10 million</b></p> <p>Encourage children to estimate and use inverse operations to check answers to calculations.</p>	
<p><b>Subtract decimals with up to 3 decimal places</b></p> <p>Progress from the same number of decimal and whole number places to a different number of decimal and whole number places.</p>	<p>I do/do not need to make an exchange because ...</p>

# Subtraction

<b>Progression of skills</b>	<b>Key representations</b>	
<p><b>Order of operations</b></p> <p>Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p>	
<p><b>Negative numbers</b></p> <p>Children subtract from positive and negative numbers and calculate intervals across 0</p>	<p>... minus ... is equal to ...</p> <p><math>-1 - 4 = -5</math></p> <p><math>1 - 4 = -3</math></p>	<p>The difference between <math>-5</math> and <math>-1</math> is 4</p> <p>The difference between 5 and <math>-5</math> is 10</p>



# Subtraction

Progression of skills	Key representations		
<p><b>Subtract fractions</b></p> <p>Convert fractions to the same denominator before subtracting. Progress from fractions where one denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.</p>	<p>The denominator has been multiplied by ..., so the numerator needs to be multiplied by...</p>  $\frac{2}{3} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9}$	<p>The lowest common multiple of ... and ... is ...</p>  $\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$	<p>... is made up of ... wholes and ...</p>  $2\frac{3}{4} - 1\frac{1}{8} = 1\frac{5}{8}$



# MULTIPLICATION & DIVISION



# Multiplication

<p><b>Reception</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> <li>• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li> </ul>
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Double to 10</b></p> <p>Prompt children to notice that double means twice as many and to notice that there are two equal groups.</p>	<p>Double ... is ... ... is double ...</p> 
<p><b>Make equal groups</b></p> <p>Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.</p>	<p>There are ... groups of ... There are ... altogether.</p> 

# Division

<p><b>Reception</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> <li>• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li> </ul>
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>
<p><b>Sharing</b></p> <p>Provide practical activities such as sharing items during snack time. Encourage children to check whether items have been shared fairly (equally).</p>	<p>There are ... altogether. They are shared equally between ... groups.</p> 
<p><b>Grouping</b></p> <p>Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.</p>	<p>There are ... groups of ... There are ... altogether.</p> 

# Year 1

## Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

## Year 1 Multiplication

### Concrete

The use of lots of use of physical objects to model the process of multiplication.



Developed into the arrangement of these displayed as arrays.

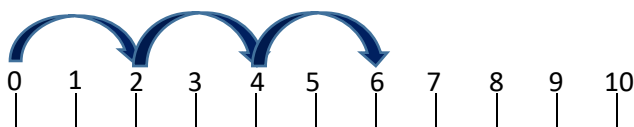


### Pictorial

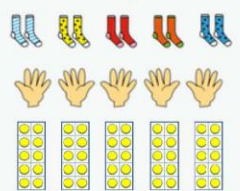

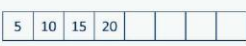

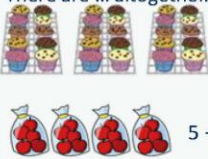
Pupils make jottings of the problems and the arrays.

Links are made with repeated addition.



$$3 \times 2 = 6$$



# Multiplication

<p><b>Year 1</b></p>	<ul style="list-style-type: none"> <li>Count in multiples of twos, fives and tens.</li> <li>Solve one-step problems involving multiplication, using concrete objects, pictorial representations and arrays with the support of the teacher.</li> </ul>		
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Count in 2s, 5s and 10s</b></p> <p>Begin by counting objects that naturally come in 2s, 5s and 10s, for example pairs of socks or fingers.</p>	<p>There are ... equal groups of ... There are ... altogether.</p> 	<p>Continue to colour in ...s What do you notice?</p> 	<p>Complete the number track/number line by counting in ...s.</p>  
<p><b>Add equal groups (repeated addition)</b></p> <p>Children should be able to write a repeated addition to represent equal groups and to draw pictures or use objects to represent a repeated addition.</p>	<p>There are ... groups of ... There are ... altogether.</p>  <p><math>10 + 10 + 10 = 30</math></p> <p><math>5 + 5 + 5 + 5 = 20</math></p>		<p>What is the same? What is different?</p> <p><math>2 + 2 + 2 =</math></p> <p><math>5 + 5 + 5 =</math></p> <p><math>10 + 10 + 10 =</math></p> <p>Use objects or a drawing to represent the equal groups and find how many in total.</p>

# Multiplication

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Make arrays</b></p> <p>Children use their knowledge of adding equal groups to arrange objects in columns and rows.</p>	<p>There are ... rows of ... There are ... altogether. There are ... columns of ... There are ... altogether.</p> 		
<p><b>Make doubles</b></p> <p>Children understand that doubles are two equal groups. Children may begin to explore doubles beyond 20 using base 10</p>	<p>Double ... is ... ... + ... = ...</p> 		

## Year 1 Division

### Concrete

*The use of lots of use of physical objects to model the process of division.*

The key difference between 'grouping' and 'sharing' in the context of division problems is explicitly modelled.



*$6 \div 2 = 3$  can be demonstrated by "grabbing" groups of 2 frogs and finding out how many lily pads are needed.*



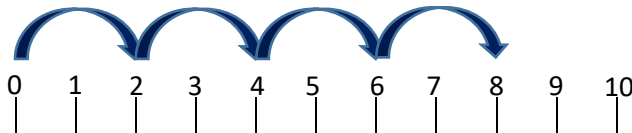
*$6 \div 2 = 3$  can be demonstrated sharing the 6 frogs equally between 2 lily pads.*

### Pictorial

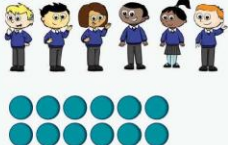


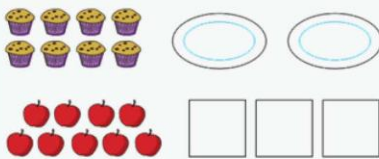

Pupils make jottings of the various problems and the arrays.

Links are made with repeated addition.


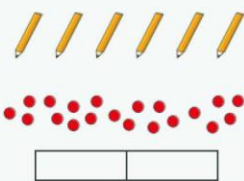
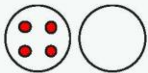
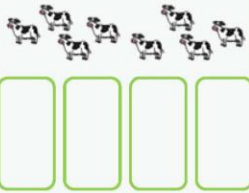
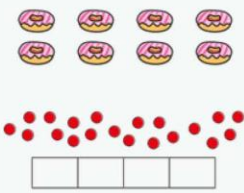
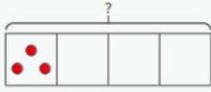
$8 \div 2 =$  "How many 2s make 8?"



# Division

<b>Year 1</b>	<ul style="list-style-type: none"> <li>Solve simple one-step problems involving division, using concrete objects, pictorial representations and arrays with the support of the teacher.</li> <li>Recognise, find and name a half as one of two equal parts of a quantity.</li> <li>Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<p><b>Make equal groups - grouping</b></p> <p>Encourage children to physically move objects into equal groups. They can also circle equal groups when using pictures.</p>	<p>There are ... altogether. How many groups of ... can you make?</p> 	<p>Circle groups of 2 There are ... groups of 2</p> 	<p>Take ... cubes. Make equal groups.</p>  <p>There are ... groups of ...</p>
<p><b>Make equal groups – sharing</b></p> <p>Encourage children to check that the objects have been shared fairly and each group is the same.</p>	<p>... have been shared equally between... There are ... on/in each ...</p> 	<p>Take ... cubes. Share them between ...</p>  <p>12 shared between ... is ...</p>	

# Division

<b>Progression of skills</b>	<b>Key representations</b>		
<p><b>Find a half</b></p> <p>Start with practical opportunities to share a quantity into 2 groups. Progress to circling half of the objects in a picture and then to finding the whole from a given half.</p>	<p>To find half, I need to share into 2 equal groups.</p>  <p>There are ... in each group.</p>	<p>Half of ... is ...</p> 	<p>If ... is half, what is the whole?</p>  <p>4 is half of ...</p>
<p><b>Find a quarter</b></p> <p>Start with practical opportunities to share a quantity into 4 groups. Progress to using pictures or bar models to find a quarter and then to finding the whole from a given quarter.</p>	<p>To find a quarter, I need to share into 4 equal groups.</p>  <p>There are ... in each group.</p>	<p>A quarter of ... is ...</p> 	<p>If ... is one quarter, what is the whole?</p>  <p>3 is one quarter of ...</p>

# Year 2

## Number – multiplication and division

### Statutory requirements

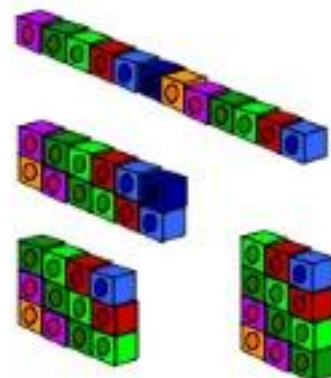
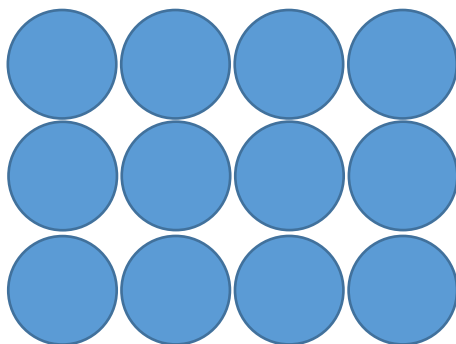
Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

### Year 2 Multiplication

#### Concrete

Develop the work from Year 1 on the use of arrays, using counters/cubes to show multiplication sentences.



#### Pictorial

Develop informal jottings to support the calculation.

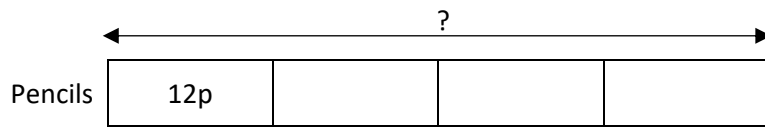
Build on the number line representation from Year 1.

$2 \times 5$  is the same as  $5 \times 2$

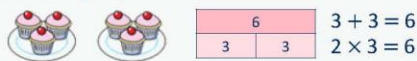
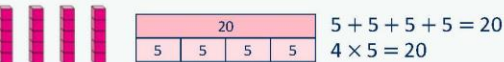



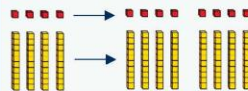


Bar modelling is introduced for multiplicative reasoning with part-whole problems:

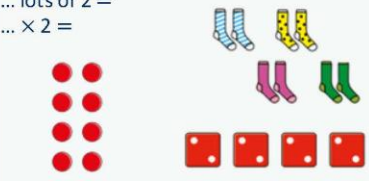


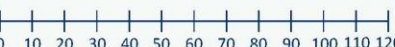
Pencils cost 12p each. How much do 4 pencils cost? (*Whole unknown*)



# Multiplication




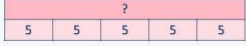



<b>Year 2</b>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (<math>\times</math>) and equals (<math>=</math>) signs.</li> <li>Show that multiplication of two numbers can be done in any order (commutative).</li> </ul>
<b>Progression of skills</b>	<b>Key representations</b>
<p><b>Link repeated addition and multiplication</b></p> <p>Encourage children to make the link between repeated addition and multiplication.</p>	<p>There are ... equal groups with ... in each group. There are ... altogether.</p>  <p>3 + 3 = 6 2 x 3 = 6</p>  <p>5 + 5 + 5 + 5 = 20 4 x 5 = 20</p>
<p><b>Use arrays</b></p> <p>Encourage children to see that multiplication is commutative.</p>	<p>There are ... rows with ... in each row. There are ... columns with ... in each column.</p>  <p>3 lots of 5 = 15 5 + 5 + 5 = 15</p>  <p>5 lots of 3 = 15 3 + 3 + 3 + 3 + 3 = 15</p> <p>I can see ... <math>\times</math> ... and ... <math>\times</math> ...</p> <p>3 x 5 = 15 5 x 3 = 15 3 x 5 = 5 x 3</p>
<p><b>Double</b></p> <p>Encourage children to make links with related facts.</p>	<p>Double ... is ...</p>  <p>Double 4 = 4 + 4 Double 4 is 8</p> <p>Double ... is ... so double ... is ...</p>  <p>Double 4 is 8 Double 40 is 80</p>

# Multiplication

<b>Progression of skills</b>	<b>Key representations</b>																																								
<p><b>The 2 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice that all multiples of 2 are even numbers.</p>	<p>... lots of 2 = ... <math>\times</math> 2 =</p>  <p>... times 2 is equal to ...</p> <table border="1" data-bbox="997 1220 1244 1288"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table> <p>1 x 2 = 2    2 = 1 x 2 2 x 2 = 4    4 = 2 x 2 3 x 2 = 6    6 = 3 x 2</p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30										
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<p><b>The 10 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.</p>	<p>... lots of 10 = ... <math>\times</math> 10 =</p>  <p>... times 10 is equal to ...</p> <table border="1" data-bbox="997 1500 1244 1601"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> </table> <p>1 x 10 = 10    10 = 1 x 10 2 x 10 = 20    20 = 2 x 10 3 x 10 = 30    30 = 3 x 10</p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
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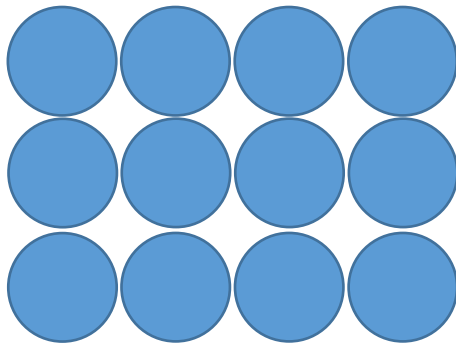
# Multiplication

Progression of skills	Key representations																																									
<p><b>The 5 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.</p>	<p>... lots of 5 =</p> <p>... <math>\times 5 =</math></p>    	<p>... times 5 is equal to ...</p> <table border="1" data-bbox="999 389 1248 488"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> </table> <p> <math>1 \times 5 = 5</math>    <math>5 = 1 \times 5</math>  <math>2 \times 5 = 10</math>    <math>10 = 2 \times 5</math>  <math>3 \times 5 = 15</math>    <math>15 = 3 \times 5</math> </p> 	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
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31	32	33	34	35	36	37	38	39	40																																	
<p><b>Missing numbers</b></p> <p>Make links to known facts.</p>	<p>... is equal to ... groups of ...</p> <p>18 socks, how many pairs? </p> 	<p>... times ... is equal to ...</p> <p><input type="text"/> <math>\times 2 = 18</math></p> <p><math>18 = 2 \times</math> <input type="text"/></p>																																								

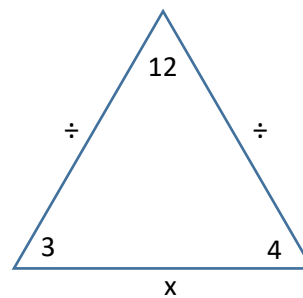
## Year 2 Division

### Concrete

With the use of counters and cubes, the link between multiplication and division is made explicit by creating an array and generating the different number sentences that can be created.

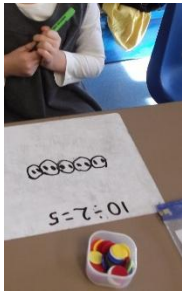


$4 + 4 + 4 = 12$	$3 \times 4 = 12$
$3 + 3 + 3 + 3 = 12$	$4 \times 3 = 12$
$12 - 4 - 4 - 4 = 0$	$12 \div 4 = 3$
$12 - 3 - 3 - 3 - 3 = 0$	$12 \div 3 = 4$

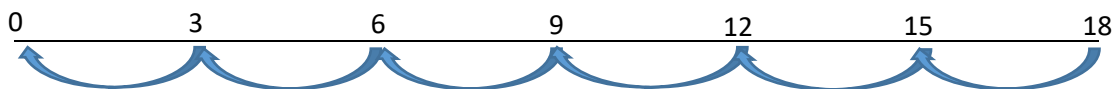


### Pictorial

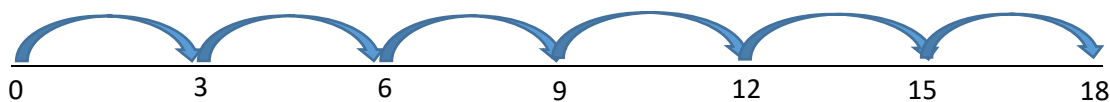
Pupils use jottings to express the division problems.



$18 \div 3$  can be modelled as sharing – 18 divided between 3 or by modelling jumping back in threes to share in 'chunks' of 3



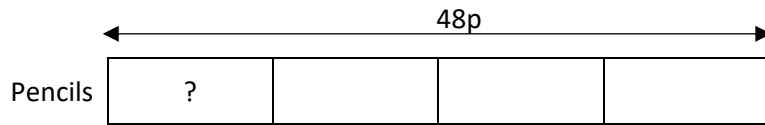
It can also be modelled in terms of grouping – how many 3s make 18?



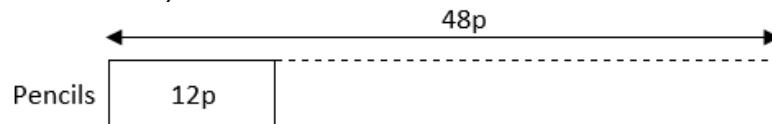
(In all calculations up to this point there are no remainders and the problems are built upon multiplication facts that the pupils are expected to be fluent in).

Bar Modelling is introduced for multiplicative (and the related divisional) reasoning:

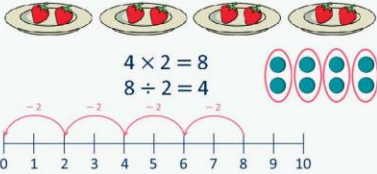
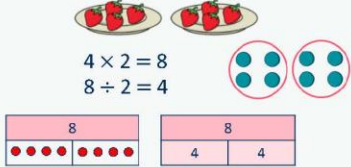
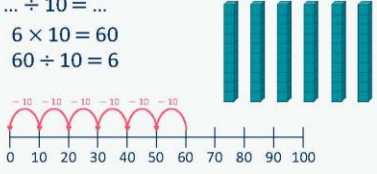
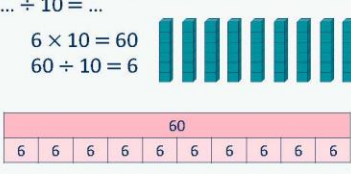
Barry bought 4 pencils for 48p. How much does 1 pencil cost? (*Value of one part unknown*)



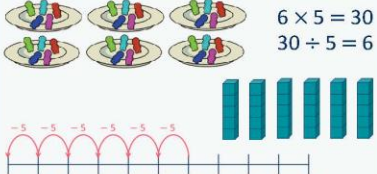
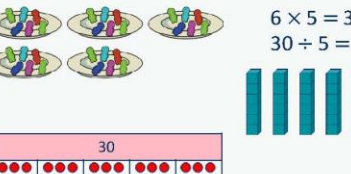
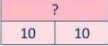
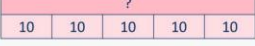
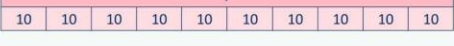
Pencils cost 12p each. David bought some pencils for 48p. How many pencils did he buy? (*Number of parts unknown*)



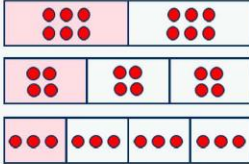

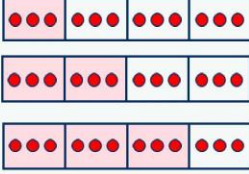

# Division

<p><b>Year 2</b></p>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for division within the multiplication tables and write them using the division (<math>\div</math>) and equals (<math>=</math>) signs.</li> <li>Recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a quantity.</li> </ul>	
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Divide by 2</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts and halving.</p>	<p>There are ... equal groups of 2 ... <math>\div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>	<p>... shared equally between 2 is ... Half of ... is ... ... <math>\div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>
<p><b>Divide by 10</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 10 ... <math>\div 10 = \dots</math></p> <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p> 	<p>... shared equally between 10 is ... ... <math>\div 10 = \dots</math></p> <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p> 

# Division

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Divide by 5</b></p> <p>Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 5 ... <math>\div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>	<p>... shared equally between 5 is ... ... <math>\div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>
<p><b>Missing numbers</b></p> <p>Bar models are useful to show the link between multiplication and division.</p>	<p>... divided by 2/5/10 is equal to ...</p> <p> <math>\square \div 2 = 10</math></p> <p> <math>\square \div 5 = 10</math></p> <p> <math>\square \div 10 = 10</math></p>	

# Division

Progression of skills	Key representations	
<p><b>Unit fractions</b></p> <p>In Y2 the focus is on finding <math>\frac{1}{2}</math>, <math>\frac{1}{4}</math> and <math>\frac{1}{3}</math></p> <p>Bar models are useful to show the link between division and finding a fraction.</p>	<p>The objects have been shared fairly into ... groups.</p> <p><math>\frac{1}{\square}</math> of ... is ...</p> 	<p>There are ... equal parts.</p> <p>There is ... part circled.</p> <p><math>\frac{1}{\square}</math> is circled.</p> 
<p><b>Non-unit fractions</b></p> <p>In Y2 the focus is on finding <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math></p> <p>Prompt children to notice that <math>\frac{2}{4}</math> is equivalent to <math>\frac{1}{2}</math></p>	<p>The objects have been shared fairly into ... groups.</p> <p><math>\frac{\square}{\square}</math> of ... is ...</p> 	<p>There are ... equal parts.</p> <p>There are ... parts circled.</p> <p><math>\frac{\square}{\square}</math> is circled.</p> 

# Year 3

## Number – multiplication and division

### Statutory requirements

Pupils should be taught to:

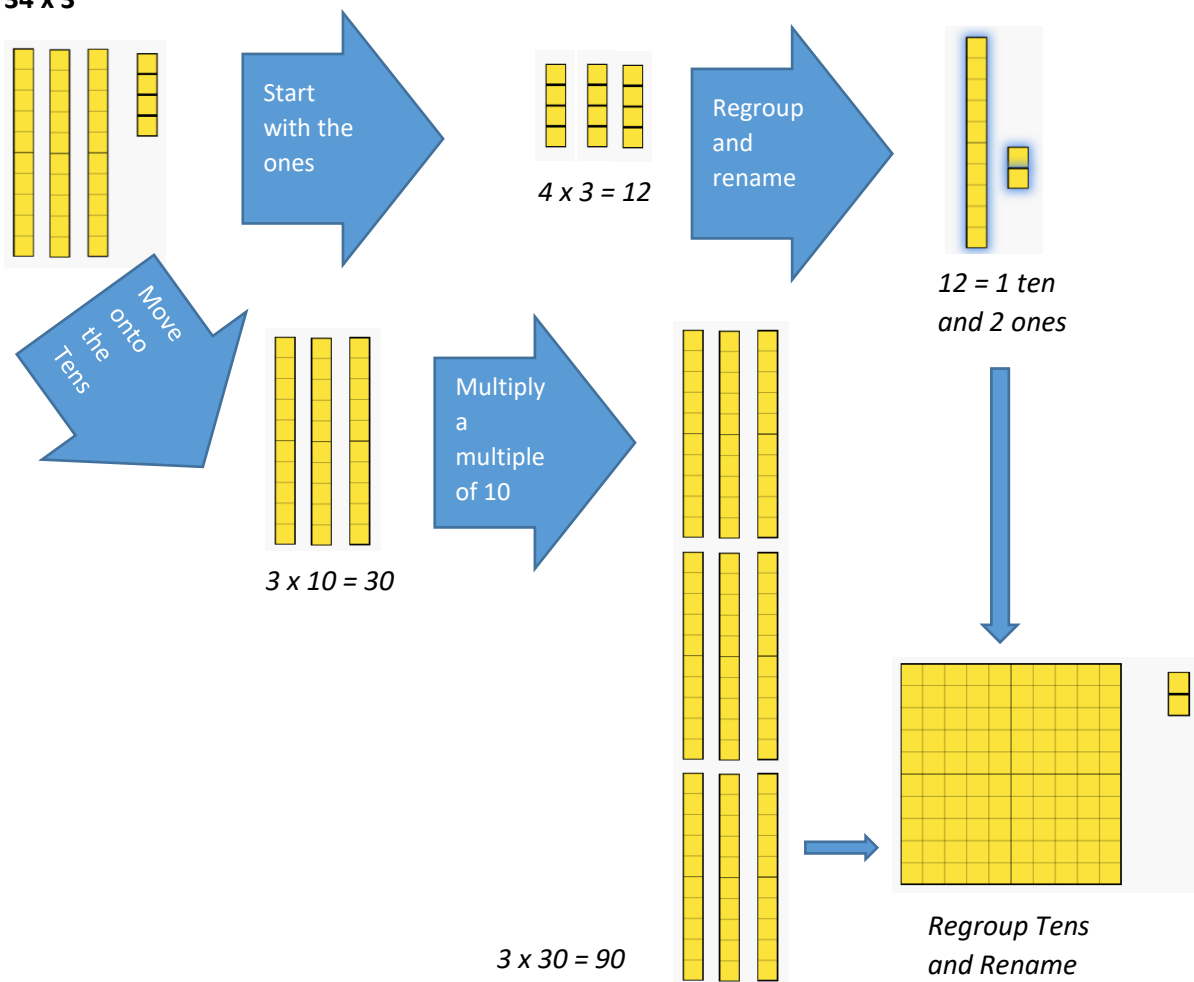
- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

### Year 3 Multiplication

#### Concrete

Base 10 materials are heavily used to model the multiplication of number.

#### 34 x 3





# Multiplication

<b>Year 3</b>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which <math>n</math> objects are connected to <math>m</math> objects.</li> </ul>																															
<b>Progression of skills</b>	<b>Key representations</b>																															
<p><b>The 3 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back.</p>	<p>... groups of 3 =</p> <p>... <math>\times 3 =</math></p> <p>3, ... times =</p> <p>3 <math>\times</math> ... =</p>	<p>... times 3 is equal to ...</p> <table border="1" style="font-size: small;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table> <p><math>4 \times 3 = 12</math>   <math>12 = 4 \times 3</math></p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
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21	22	23	24	25	26	27	28	29	30																							
<p><b>The 4 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2 and 4 times-tables.</p>	<p>... groups of 4 =</p> <p>... <math>\times 4 =</math></p> <p>4, ... times =</p> <p>4 <math>\times</math> ... =</p>	<p>... times 4 is equal to ...</p> <table border="1" style="font-size: small;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table> <p><math>3 \times 4 = 12</math>   <math>12 = 3 \times 4</math></p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
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11	12	13	14	15	16	17	18	19	20																							
21	22	23	24	25	26	27	28	29	30																							

# Multiplication

<b>Progression of skills</b>	<b>Key representations</b>																															
<p><b>The 8 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2, 4 and 8 times-tables.</p>	<p>... lots of 8 =</p> <p><math>\times 8 =</math></p> <p>8, ... times =</p> <p>8 <math>\times</math> ... =</p>	<p>... times 8 is equal to ...</p> <table border="1" style="font-size: small;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </table> <p><math>3 \times 8 = 24</math>   <math>24 = 3 \times 8</math></p>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
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11	12	13	14	15	16	17	18	19	20																							
21	22	23	24	25	26	27	28	29	30																							
<p><b>Related facts</b></p> <p>Use knowledge of multiplying by 10 to scale times-table facts.</p>	<p>... <math>\times</math> ... ones is equal to ... ones so ... <math>\times</math> ... tens is equal to ... tens.</p> <p><math>3 \times 4 = 12</math> <math>3 \times 40 = 120</math></p>																															
<p><b>Multiply a 2-digit number by a 1-digit number - no exchange</b></p> <p>Children apply their understanding of partitioning to represent and solve calculations using the expanded method.</p>	<p>... tens multiplied by ... is equal to ... tens. ...ones multiplied by ... is equal to ... ones.</p> <table border="1" style="font-size: small;"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td>30</td><td>2</td></tr> <tr><td>20</td><td>2</td></tr> </table> <p><math>30 \times 2 = 60</math> <math>2 \times 2 = 4</math> <math>32 \times 2 = 64</math></p> <table border="1" style="font-size: small;"> <tr><th>Tens</th><th>Ones</th></tr> <tr><td>60</td><td>4</td></tr> <tr><td>60</td><td>4</td></tr> <tr><td>60</td><td>4</td></tr> <tr><td>60</td><td>4</td></tr> </table>		Tens	Ones	30	2	20	2	Tens	Ones	60	4	60	4	60	4	60	4														
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# Multiplication

Progression of skills	Key representations																		
<p><b>Multiply a 2-digit number by a 1-digit number - with exchange</b></p> <p>Children apply their understanding of partitioning to represent and solve calculations using the expanded method.</p>	<p>... tens multiplied by ... is equal to ... tens. ... ones multiplied by ... is equal to ... ones.</p> <table border="1"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table> <p><math>20 \times 4 = 80</math> <math>4 \times 4 = 16</math> <math>24 \times 4 = 96</math></p> <p><math>45 \times 3</math></p> <table border="1"> <thead> <tr> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td></td> </tr> </tbody> </table> <p><math>40 \times 3</math>   <math>5 \times 3</math></p>	Tens	Ones									Tens	Ones						
Tens	Ones																		
Tens	Ones																		
<p><b>Scaling</b></p> <p>Children focus on multiplication as scaling ( ... times the size) as opposed to repeated addition.</p>	<p>There are ... times as many ... as ...</p> <p></p> <p>There are 3 times as many triangles as circles.</p> <p>... is ... times the size of ... ... is ... times the length/height of ...</p> <p></p> <p>Miss Smith is twice the height of Jo.</p>																		

# Multiplication

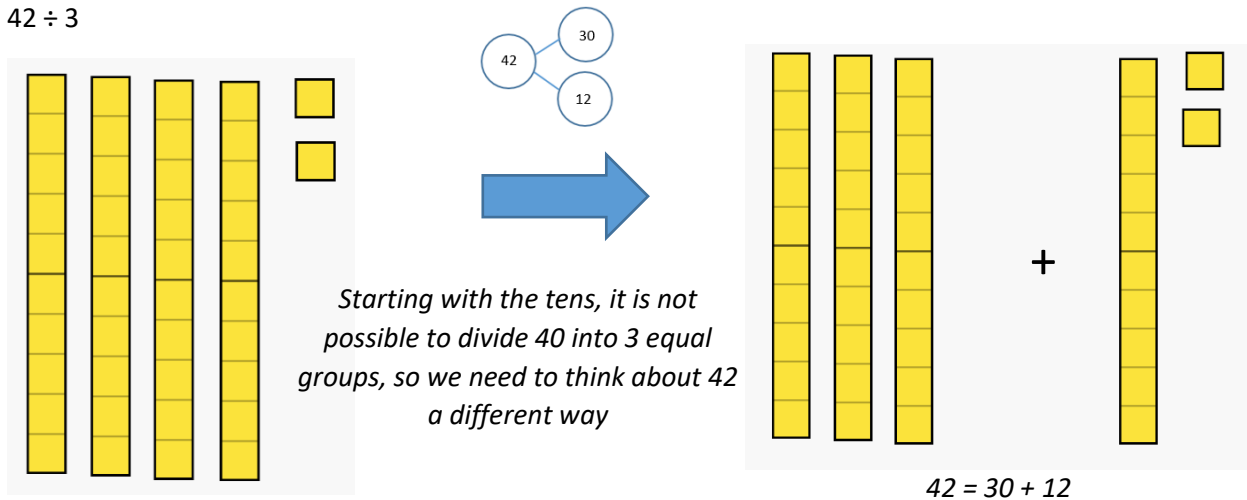
Progression of skills	Key representations												
<p><b>Correspondence problems (How many ways?)</b></p> <p>Encourage children to work systematically to find all the different possible combinations.</p>	<p>For every ... , there are ... possible ... There are ... <math>\times</math> ... possibilities altogether.</p> <table border="1"> <thead> <tr> <th></th> <th>hats</th> <th>scarves</th> </tr> </thead> <tbody> <tr> <th>blue</th> <td></td> <td></td> </tr> <tr> <th>orange</th> <td></td> <td></td> </tr> <tr> <th>purple</th> <td></td> <td></td> </tr> </tbody> </table> <p>For every hat, there are two possible scarves. <math>3 \times 2 = 6</math></p> <p>There are 6 possibilities altogether.</p>		hats	scarves	blue			orange			purple		
	hats	scarves											
blue													
orange													
purple													

## Year 3 Division

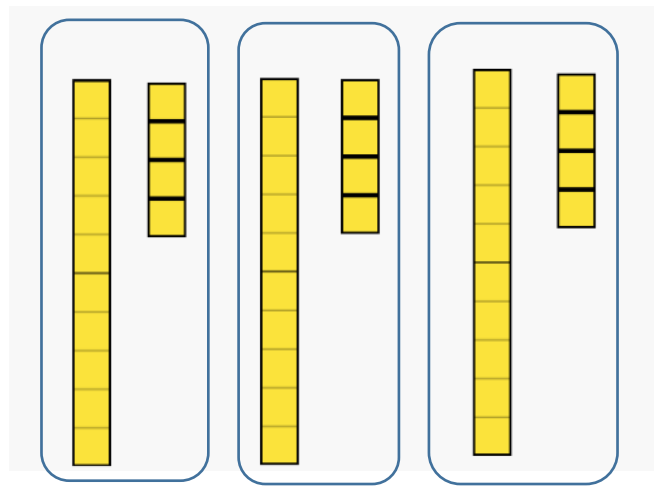
### Concrete

The concrete resources from lower down the school continue to be used to demonstrate the difference between grouping and sharing. As with multiplication, extensive use is made of Base 10 materials to demonstrate what is happening with the division.

$$42 \div 3$$



The tens can now be divided into 3 equal groups. The 12 ones can also be equally divided into 3 groups:

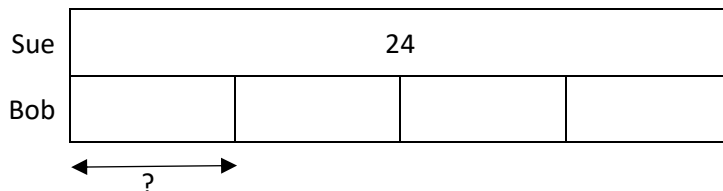


So 42 divided by 3 results in 3 equal groups of 14.

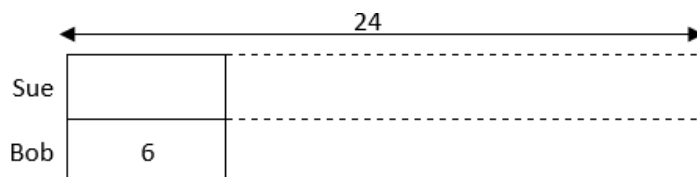
### Pictorial

The mastery of multiplicative (and related divisional) reasoning problems from KS1 is checked, with revision as necessary. Develop use of bar model with comparing problems:

Sue picked 24 apples. She picked four times as many apples as Bob. How many apples did Bob pick? (*Smaller quantity unknown*)



Sue picked 24 apples. Bob picked 6 apples. How many more times as many apples did Sue pick than Bob? (*Multiplier unknown*)

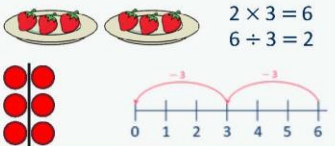
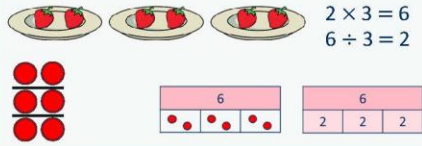
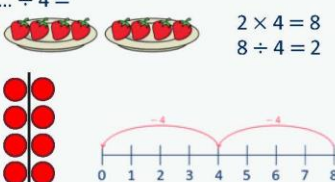
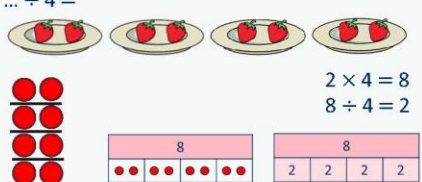


### Abstract

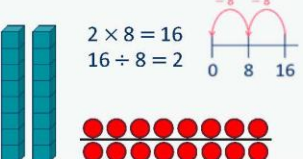
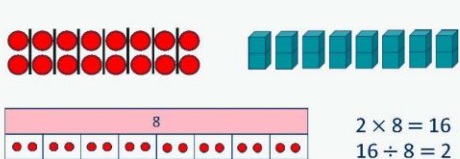
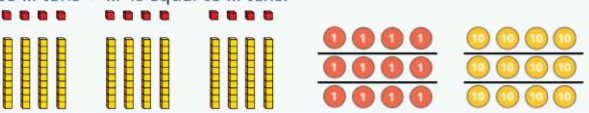
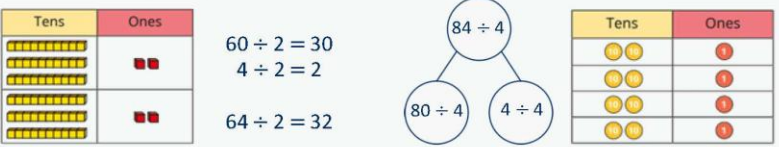
With the links between the concrete representation (using Base 10 materials) very clear, modelling the process side by side.

$$\begin{array}{r} 14 \\ 3 \overline{) 42} \\ - 30 \quad (10x) \\ \hline 12 \\ - 12 \quad (4x) \\ \hline 0 \end{array}$$

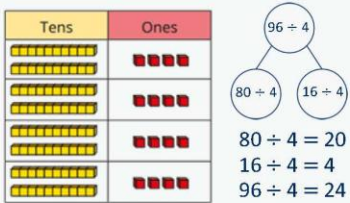
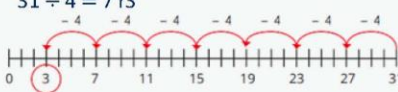
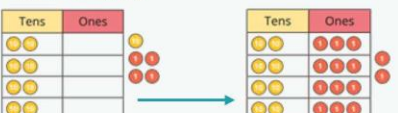
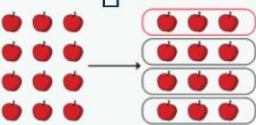


# Division

<b>Year 3</b>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<b>Divide by 3</b>  Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are ... groups of 3 in ... $\dots \div 3 =$  $2 \times 3 = 6$ $6 \div 3 = 2$	... has been shared equally into 3 equal groups. $\dots \div 3 =$  $2 \times 3 = 6$ $6 \div 3 = 2$
<b>Divide by 4</b>  Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are ... groups of 4 in ... $\dots \div 4 =$  $2 \times 4 = 8$ $8 \div 4 = 2$	... has been shared equally into 4 equal groups. $\dots \div 4 =$  $2 \times 4 = 8$ $8 \div 4 = 2$

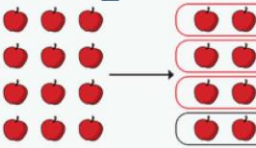


# Division

<b>Progression of skills</b>	<b>Key representations</b>	
<b>Divide by 8</b>  Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are ... groups of 8 in ... $\dots \div 8 =$  $2 \times 8 = 16$ $16 \div 8 = 2$	... has been shared equally into 8 equal groups. $\dots \div 8 =$  $2 \times 8 = 16$ $16 \div 8 = 2$
<b>Related facts</b>  Link to known times-table facts.	... $\div$ ... is equal to ..., so ... tens $\div$ ... is equal to ... tens.  $12 \div 3 = 4$ $120 \div 3 = 40$	
<b>Divide a 2-digit number by a 1-digit number - no exchange</b>  Partition into tens and ones to divide and then recombine.	... tens divided by ... is equal to ... tens. ... ones divided by ... is equal to ... ones.  $60 \div 2 = 30$ $4 \div 2 = 2$ $64 \div 2 = 32$ $84 \div 4$ $80 \div 4$ $4 \div 4$	

# Division

Progression of skills	Key representations	
<p><b>Divide a 2-digit number by a 1-digit number - with remainders</b></p> <p>Encourage children to partition numbers flexibly to help them to divide more efficiently.</p>	<p>... tens divided by ... is equal to ... tens. ... ones divided by ... is equal to ... ones.</p>  <p><math>96 \div 4</math></p> <p><math>80 \div 4 = 20</math> <math>16 \div 4 = 4</math> <math>96 \div 4 = 24</math></p>	<p>There are ... groups of ... There are ... remaining.</p> <p><math>31 \div 4 = 7 \text{ r}3</math></p>  <p><math>94 \div 4 = 23 \text{ r}2</math></p> 
<p><b>Unit fractions of a set of objects</b></p> <p>Bar models are useful to show the link between division and fractions, for example, dividing by 3 and finding a third.</p>	<p>The whole is divided into ... equal parts. Each part is <math>\frac{1}{\square}</math> of the whole.</p>  <p><math>\frac{1}{4}</math> of 12 apples is 3 apples.</p>	<p>One ... of ... is ...</p> <p><math>\frac{1}{4}</math> of 12 is 3</p>  <p><math>\frac{1}{3}</math> of 36 is 12</p> 

# Division

Progression of skills	Key representations	
<p><b>Non-unit fractions of a set of objects</b></p> <p>Bar models are a useful representation and show the links with division and multiplication.</p>	<p>The whole is divided into ... equal parts. Each part is <math>\frac{1}{\square}</math> of the whole.</p>  <p><math>\frac{3}{4}</math> of 12 apples is 9 apples.</p>	<p><math>\frac{1}{\square}</math> of ... is ..., so <math>\frac{\square}{\square}</math> of ... is ...</p> <p><math>\frac{3}{4}</math> of 12 is 9</p>  <p><math>\frac{2}{3}</math> of 36 is 24</p> 

# Year 4

## Number – multiplication and division

### Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

### Year 4 Multiplication

#### Pictorial

Bar Model approaches introduced in KS1 and Year 3 for multiplicative reasoning (and related divisional problems) are used and revised as necessary.

#### Abstract

Continue to model concrete and visual representations practically alongside the written calculations which are more formally introduced in Year 4.


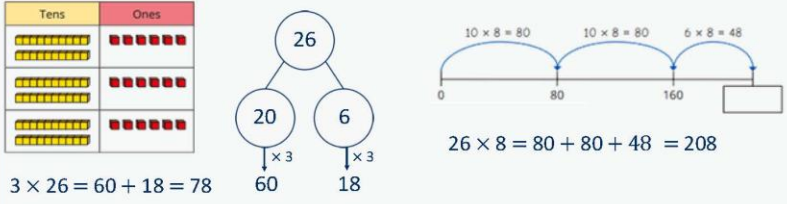
Calculations follow the 'expanded' formal written method:

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 1 \quad 4 \quad 3 \\ \times \quad \quad 6 \\ \hline \quad \quad 1 \quad 8 \quad (3 \times 6) \\ \quad 2 \quad 4 \quad 0 \quad (40 \times 6) \\ 6 \quad 0 \quad 0 \quad (100 \times 6) \\ \hline 8 \quad 5 \quad 8 \end{array}$$

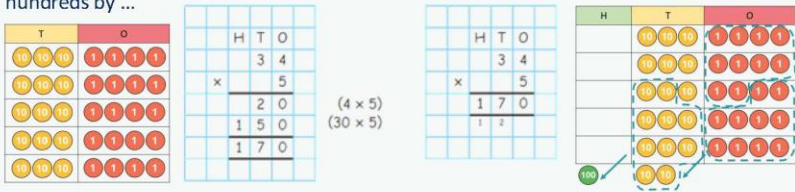

Derived from the knowledge that as  $4 \times 6 = 24$  then  $40 \times 6$  would be 10 times greater.



# Multiplication

Progression of skills	Key representations
<p><b>Related facts</b></p> <p>Use knowledge of multiplying by 10 and 100 to scale times-table facts.</p>	<p>... × ... ones is equal to ... ones so ... × ... tens is equal to ... tens and ... × ... hundreds is equal to ... hundreds.</p>  <p> <math>3 \times 7 = 21</math>      <math>7 \times 3 = 21</math>  <math>3 \times 70 = 210</math>      <math>7 \times 30 = 210</math>  <math>3 \times 700 = 2,100</math>      <math>7 \times 300 = 2,100</math> </p>
<p><b>Mental strategies</b></p> <p>Partition 2 or 3-digit numbers to multiply using informal methods.</p>	<p>... tens multiplied by ... is equal to ... tens. ...ones multiplied by ... is equal to ... ones.</p>  <p> <math>3 \times 26 = 60 + 18 = 78</math> </p> <p> <math>26 \times 8 = 80 + 80 + 48 = 208</math> </p>

# Multiplication

Progression of skills	Key representations																								
<p><b>Multiply a 2 or 3-digit number by a 1-digit number</b></p> <p>The short multiplication method is introduced for the first time, initially in an expanded form.</p>	<p>To multiply a 2-digit number by ... , I multiply the ones by ... and the tens by ... To multiply a 3-digit number by ... , I multiply the ones by ... , the tens by ... and the hundreds by ...</p>  <p> <math>(4 \times 5)</math>  <math>(30 \times 5)</math> </p>																								
<p><b>Scaling</b></p> <p>Children focus on multiplication as scaling (... times the size).</p>	<p>... is ... times the size of ...</p>  <p>         A computer mouse costs £7          A keyboard costs 6 times as much.     </p> <p>         A red ribbon is 6 cm.          A yellow ribbon is 7 times as long.     </p>																								
<p><b>Correspondence problems</b></p> <p>Encourage children to use tables to show all the different possible combinations.</p>	<p>For every ... , there are ... possibilities. There are ... × ... possibilities altogether.</p> <p>A pizza company offers a choice of 5 toppings and 3 bases.</p> <p><math>5 \times 3 = 15</math></p> <table border="1" data-bbox="885 1646 1324 1792"> <thead> <tr> <th></th> <th>Deep pan</th> <th>Italian</th> <th>Thin</th> </tr> </thead> <tbody> <tr> <th>Cheese</th> <td>C DP</td> <td>C I</td> <td>C Th</td> </tr> <tr> <th>Mushroom</th> <td>M DP</td> <td>M I</td> <td>M Th</td> </tr> <tr> <th>Vegetable</th> <td>V DP</td> <td>V I</td> <td>V Th</td> </tr> <tr> <th>Chicken</th> <td>C DP</td> <td>C I</td> <td>C Th</td> </tr> <tr> <th>Tuna</th> <td>T DP</td> <td>T I</td> <td>T Th</td> </tr> </tbody> </table>		Deep pan	Italian	Thin	Cheese	C DP	C I	C Th	Mushroom	M DP	M I	M Th	Vegetable	V DP	V I	V Th	Chicken	C DP	C I	C Th	Tuna	T DP	T I	T Th
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Cheese	C DP	C I	C Th																						
Mushroom	M DP	M I	M Th																						
Vegetable	V DP	V I	V Th																						
Chicken	C DP	C I	C Th																						
Tuna	T DP	T I	T Th																						

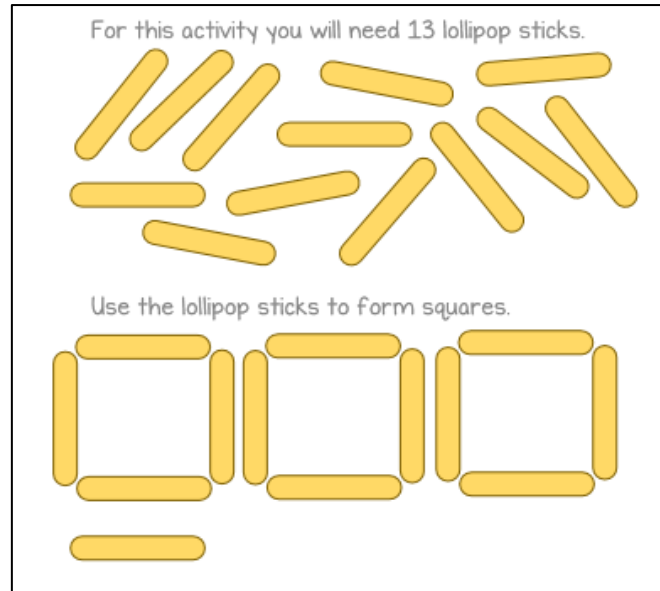


## Year 4 Division

### Concrete

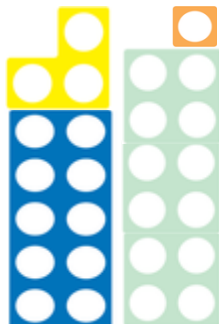
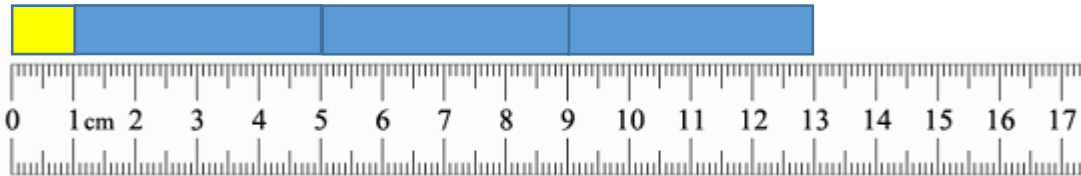
As the introduction of the concept of remainders with division is introduced, there is an increased use of concrete resources to reinforce this learning.

For example, using lollipop sticks to investigate a sum such as  $13 \div 4$



$13 \div 4$  can be represented as 13 lollipop sticks make 4 squares with one stick left over ( $13 \div 4 = 3 \text{ r}1$ )

This would also be modelled with counters/objects as lower in the school as well as

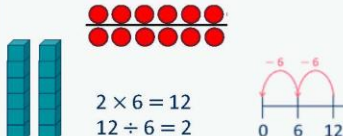
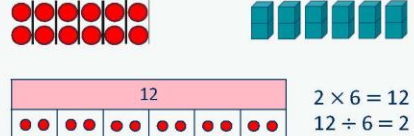






### Pictorial


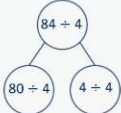
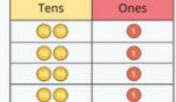
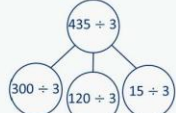

Bar Model approaches introduced in KS1 and Year 3 for multiplicative reasoning (and related divisional problems) are used and revised as necessary.



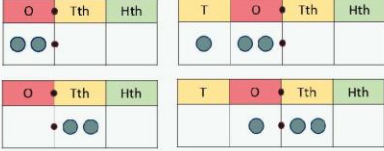
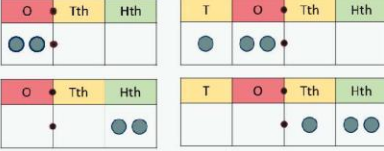
# Division

<b>Year 4</b>	<ul style="list-style-type: none"> <li>Recall division facts for multiplication tables up to <math>12 \times 12</math></li> <li>Use place value, known and derived facts to divide mentally, including: dividing by 1</li> <li>Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<b>Division facts to <math>12 \times 12</math></b>  Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are ... groups of ... in ... $\dots \div \dots =$  	... has been shared equally into ... equal groups. $\dots \div \dots =$  
<b>Divide a number by 1 and itself</b>  Children may try to divide a number by zero and it should be highlighted that this is not possible.	When I divide a number by 1, the number remains the same.  5 shared between 1 is 5   There are 5 groups of 1 in 5 	When I divide a number by itself, the answer is 1  5 shared between 5 is 1   There is 1 group of 5 in 5 

# Division

<b>Progression of skills</b>	<b>Key representations</b>	
<b>Related facts</b>  Link to known times-table facts.	$\dots \div \dots$ is equal to ... so ... tens $\div$ ... is equal to ... tens and ... hundreds $\div$ ... is equal to ... hundreds.  	
<b>Divide a 2 or 3-digit number by a 1-digit number</b>  Progress from divisions with no exchange, to divisions with exchange and then divisions with remainders.	I can partition ... into ... tens and ... ones.   $80 \div 4 = 20$ $4 \div 4 = 1$ $84 \div 4 = 21$  	I cannot share the hundreds/tens equally, so I need to exchange 1 ... for 10 ...   $300 \div 3 = 100$ $120 \div 3 = 40$ $15 \div 3 = 5$ $435 \div 3 = 145$  

# Division

Progression of skills	Key representations	
<p><b>Divide by 10 and 100</b></p> <p>Encourage children to notice that dividing by 100 is the same as dividing by 10 twice.</p>	<p>When I divide by 10, the digits move 1 place value column to the right. ... is one-tenth the size of ...</p>  <p><math>2 \div 10 = 0.2</math>      <math>12 \div 10 = 1.2</math></p>	<p>When I divide by 100, the digits move 2 place value columns to the right. ... is one-hundredth the size of ...</p>  <p><math>2 \div 100 = 0.02</math>      <math>12 \div 100 = 0.12</math></p>

## Year 5

### Number – multiplication and division

#### Statutory requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared ( $^2$ ) and cubed ( $^3$ )
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

#### Year 5 Multiplication

##### Abstract

Continue to model concrete and visual representations practically alongside the written calculations.

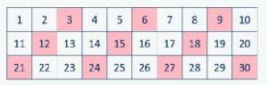

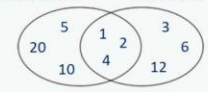
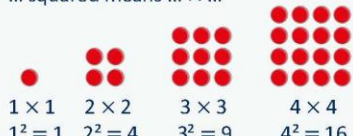
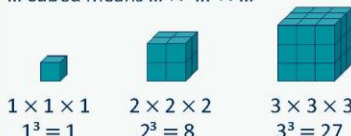
$$\begin{array}{r}
 \text{H T O} \\
 143 \\
 \times \quad \quad 6 \\
 \hline
 858 \\
 \hline
 \end{array}$$

Developing later in the year to 2 digit by 2 digit calculations.

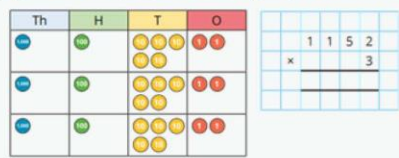
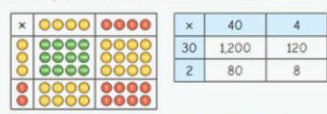
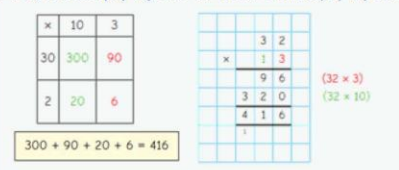
$$\begin{array}{r}
 \text{T O} \\
 37 \\
 \times \quad 29 \\
 \hline
 333 \quad (\times 9) \\
 \hline
 740 \quad (\times 20) \\
 \hline
 1073 \\
 \hline
 \end{array}$$

The zero is added at the beginning to remind that the calculation is  $\times 20$  (therefore 10 times bigger than  $\times 2$ )

# Multiplication

<b>Year 5</b>	<ul style="list-style-type: none"> <li>Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers</li> <li>Recognise and use square numbers and cube numbers, and the notation for squared (<sup>2</sup>) and cubed (<sup>3</sup>)</li> <li>Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.</li> <li>Multiply numbers mentally drawing upon known facts.</li> <li>Multiply whole numbers and those involving decimals by 10, 100 and 1000</li> <li>Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.</li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Multiples and factors</b>  Encourage children to notice patterns and make links with known facts.	... is a multiple of ... because $\dots \times \dots = \dots$ 	... is a factor of ... because $\dots \times \dots = \dots$  <p>1, 2, 4 and 8 are factors of 8</p>	The common factors of ... and ... are ...  Factors of 20: 1, 2, 4, 5, 10, 20 Factors of 12: 1, 2, 3, 4, 6, 12 
<b>Square and cube numbers</b>	... squared means $\dots \times \dots$  <p> <math>1 \times 1 = 1</math>   <math>2 \times 2 = 4</math>   <math>3 \times 3 = 9</math>   <math>4 \times 4 = 16</math>  <math>1^2 = 1</math>   <math>2^2 = 4</math>   <math>3^2 = 9</math>   <math>4^2 = 16</math> </p>	... cubed means $\dots \times \dots \times \dots$  <p> <math>1 \times 1 \times 1 = 1</math>   <math>2 \times 2 \times 2 = 8</math>   <math>3 \times 3 \times 3 = 27</math>  <math>1^3 = 1</math>   <math>2^3 = 8</math>   <math>3^3 = 27</math> </p>	

# Multiplication

<b>Progression of skills</b>	<b>Key representations</b>	
<b>Multiply numbers up to 4 digits by a 1-digit number</b>  This builds on the short multiplication method introduced in Y4	To multiply a 4-digit number by ..., I multiply the ones by ..., the tens by ..., the hundreds by ... and the thousands by ...  	
<b>Multiply numbers up to 4 digits by a 2-digit number</b>  Numbers are first partitioned using an area model then long multiplication is introduced for the first time.	I can partition ... into ... and ...  <p> <math>32 \times 44 = 1,200 + 80 + 120 + 8</math>  <math>32 \times 44 = 1,408</math> </p>	First, I multiply by the ... Then I multiply by the ...  <p> <math>300 + 90 + 20 + 6 = 416</math> </p>

# Multiplication

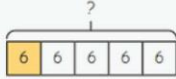
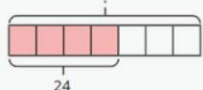
Progression of skills	Key representations																										
<p><b>Multiply by 10, 100 and 1,000</b></p> <p>Some children may over-generalise that multiplying by a power of 10 always results in adding zeros. This will cause issues later when multiplying decimals.</p>	<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><th>M</th><th>HTh</th><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td></td><td></td><td></td><td></td><td>●●</td><td>●●●</td><td>●●●</td></tr> </table> <table border="1" style="display: inline-table;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th><th>TTh</th><th>HTh</th></tr> <tr><td></td><td></td><td></td><td>●●</td><td>●●●</td><td>●●●</td></tr> </table> <p> <math>234 \times 10 = 2,340</math>  <math>234 \times 100 = 23,400</math>  <math>234 \times 1,000 = 234,000</math> </p> <p> <math>2.34 \times 10 = 23.4</math>  <math>2.34 \times 100 = 234</math>  <math>2.34 \times 1,000 = 2,340</math> </p>	M	HTh	TTh	Th	H	T	O					●●	●●●	●●●	Th	H	T	O	TTh	HTh				●●	●●●	●●●
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				●●	●●●	●●●																					
Th	H	T	O	TTh	HTh																						
			●●	●●●	●●●																						
<p><b>Mental strategies</b></p> <p>Children continue to use efficient mental strategies such as partitioning and knowledge of factor pairs and related facts to multiply.</p>	<p>The most efficient strategy to calculate ... <math>\times</math> ... is ... To calculate ... <math>\times</math> 12, I can do ... <math>\times</math> ... <math>\times</math> ...</p> <p>For example: <math>121 \times 12</math>  I could calculate <math>100 \times 12</math> plus <math>20 \times 12</math> plus <math>1 \times 12</math>  I could calculate <math>121 \times 10</math> plus <math>121 \times 2</math>  I could calculate <math>121 \times 6 \times 2</math>  I could calculate <math>121 \times 4 \times 3</math></p>																										

# Multiplication

Progression of skills	Key representations																							
<p><b>Multiply fractions by a whole number</b></p> <p>Make links with repeated addition. E.g. <math>\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}</math></p>	<p>To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.</p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td><math>\frac{1}{7}</math></td><td></td><td></td></tr> </table> <p><math>\frac{1}{7} \times 5 = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p><math>\frac{2}{7} \times 3 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td><td><math>\frac{1}{5}</math></td></tr> </table> <p><math>\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}</math></p> <table border="1" style="display: inline-table;"> <tr><td><math>\frac{2}{5}</math></td><td><math>\frac{2}{5}</math></td><td><math>\frac{2}{5}</math></td></tr> </table> <p><math>\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}</math></p>	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$										$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
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$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$																			
$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$																						
<p><b>Multiply mixed numbers by a whole number</b></p>	<p>I can partition <math>\frac{2}{3}</math> into <math>\frac{2}{3}</math> and <math>\frac{2}{3}</math></p> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td><math>\frac{2}{3}</math></td><td><math>\frac{2}{3}</math></td><td><math>\frac{2}{3}</math></td></tr> </table> <p><math>2\frac{2}{3} \times 3</math>  <math>2 \times 3 = 6</math>     <math>\frac{2}{3} \times 3 = \frac{6}{3} = 2</math></p> <p><math>2\frac{2}{3} \times 3 = 6 + 2 = 8</math></p>	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$																				
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$																						



# Multiplication

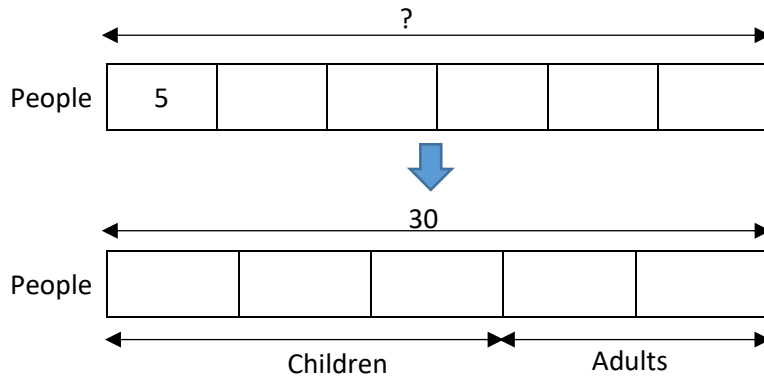
Progression of skills	Key representations	
<p><b>Find the whole</b></p> <p>Children multiply to find the whole from a given part.</p>	<p>If <math>\frac{1}{5}</math> is ... , then the whole is ... <math>\times</math> ...</p> <p><math>\frac{1}{5}</math> of <math>\underline{\quad}</math> = 6</p>  <p><math>5 \times 6 = 30</math></p> <p><math>\frac{1}{5}</math> of <b>30</b> = 6</p>	<p>If <math>\frac{4}{7}</math> is ... , then <math>\frac{1}{7}</math> is ... and the whole is ... <math>\times</math> ...</p> <p><math>\frac{4}{7}</math> of <math>\underline{\quad}</math> = 24</p> <p><math>\frac{1}{7} = 24 \div 4 = 6</math></p> <p><math>7 \times 6 = 42</math></p> <p><math>\frac{4}{7}</math> of <b>42</b> = 24</p> 

## Year 5 Division

### Pictorial

Check on retention of use of bar modelling for part-part-whole and comparing problems from Year 3 and Year 4, revising as necessary. Develop use of bar models with multi-step part-whole problems:

There are 5 people living in each of the 6 houses on Green Street.  $\frac{3}{5}$  of these people are children and the rest are adults. How many adults live on Green Street?



### Abstract

Continue to model and reinforce with concrete resources and visual representations throughout in order that pupils understand what the written strategies represent.

Pupils apply the short division strategy (bus stop method)

$$\begin{array}{r} 196 \\ 6 \overline{) 115736} \end{array}$$

Extend this to deal with a remainder:

$$\begin{array}{r} 196 \text{ r}3 \\ 6 \overline{) 115739} \end{array}$$

Pupils continue to make jottings to help with the calculations:

6  
12  
18  
24  
30  
36  
42  
48  
54  
60

Extend further to interpret the remainder as a fraction ( $\frac{3}{6}$ ) and then as a decimal:

$$\begin{array}{r} 196.5 \\ 6 \overline{) 115739.30} \end{array}$$

# Division

<b>Year 5</b>	<ul style="list-style-type: none"> <li>Divide numbers mentally drawing upon known facts.</li> <li>Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.</li> <li>Divide whole numbers and those involving decimals by 10, 100 and 1,000</li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Mental strategies</b>	<p>I can partition ... into ... and ... to help me to divide more easily.</p>	<p>I can show groups of ... on a number line.</p>	<p>To divide by ..., I can divide by ... and then divide the result by ...</p> $436 \div 4 = 436 \div 2 \div 2$ $436 \div 2 = 218$ $218 \div 2 = 109$
<b>Divide numbers up to 4 digits by a 1-digit number</b>	<p>There are ... groups of ... hundreds/tens/ones/ in ... I can exchange 1 ... for 10 ...</p>		

# Division

<b>Progression of skills</b>	<b>Key representations</b>	
<b>Divide by 10, 100 and 1,000</b>	<p>To divide by 10/100/1,000, I move all the digits ... places to the right. ... is one-tenth/one-hundredth/one-thousandth the size of ...</p> <p><math>120 \div 10 = 12</math></p> <p><math>120 \div 100 = 1.2</math></p> <p><math>120 \div 1,000 = 0.12</math></p>	
<b>Fraction of an amount</b>	<p>To find <math>\frac{\square}{\square}</math> of ..., I need to divide by ... and multiply by ...</p> <p><math>\frac{1}{5}</math> of 20 =</p> <p><math>\frac{3}{5}</math> of 20 =</p> <p><math>\frac{1}{4}</math> of 84 =</p> <p><math>\frac{3}{4}</math> of 84 =</p>	<p>If <math>\frac{1}{\square}</math> is ..., then the whole is ... <math>\times</math> ...</p> <p><math>\frac{1}{5}</math> of <math>\underline{\quad}</math> = 6</p> <p><math>\frac{4}{7}</math> of <math>\underline{\quad}</math> = 24</p>

## Year 6

### Number – multiplication and division

#### Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

#### Year 6 Multiplication

##### Abstract

Continue to model concrete and visual representations practically alongside the written calculations.

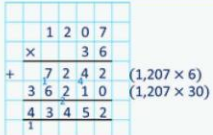
2314 x 23

	Th	H	T	O	
	2	3	1	4	
x			2	3	
<hr/>					
	6	9	4	2	(x3)
			<i>1</i>		
	4	6	2	8	0 (x20)
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	5	3	2	2	2
	<i>1</i>	<i>1</i>	<i>1</i>		

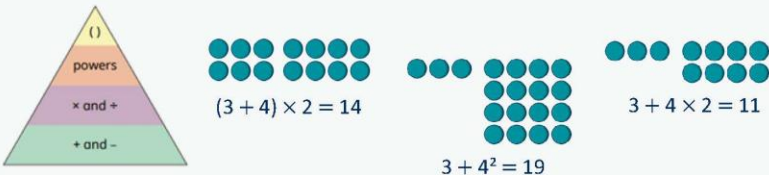

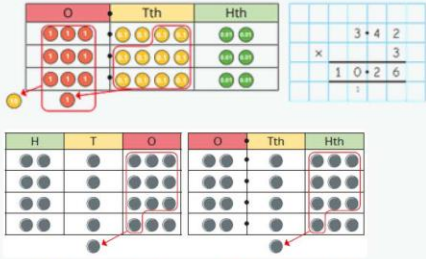
Progress onto calculations involving decimals:

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \quad \frac{1}{10} \\ \quad \quad 3 \quad 6 \cdot 2 \\ \times \quad \quad \quad 7 \\ \hline 2 \quad 5 \quad 3 \cdot 4 \\ \hline \cancel{2} \quad \cancel{4} \quad \cancel{1} \end{array}$$


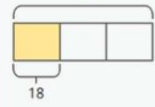
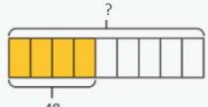
# Multiplication

<p><b>Year 6</b></p>	<ul style="list-style-type: none"> <li>Identify common factors and common multiples.</li> <li>Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</li> <li>Multiply numbers by 10, 100 and 1,000</li> <li>Multiply one-digit numbers with up to two decimal places by whole numbers.</li> <li>Use their knowledge of the order of operations to carry out calculations involving the 4 operations.</li> <li>Multiply simple pairs of proper fractions, writing the answer in its simplest form.</li> <li>Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.</li> <li>Solve problems involving the calculation of percentages.</li> </ul>																												
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>																												
<p><b>Multiply numbers up to 4 digits by a 2-digit number</b></p>	<p>To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.</p> 																												
<p><b>Multiply by 10, 100 and 1,000</b> Some children may over-generalise that multiplying by a power of 10 always results in adding zeros.</p>	<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> <table border="1" data-bbox="531 831 1289 902"> <tr> <td>M</td><td>Hth</td><td>Tth</td><td>Th</td><td>H</td><td>T</td><td>O</td><td>Th</td><td>H</td><td>T</td><td>O</td><td>Tth</td><td>Hth</td><td>Thth</td> </tr> <tr> <td></td><td></td><td></td><td></td><td>●●</td><td>●●</td><td>●●</td><td></td><td></td><td></td><td></td><td>●●</td><td>●●</td><td>●●</td> </tr> </table> <p> <math>234 \times 10 = 2,340</math>      <math>0.234 \times 10 = 2.34</math>  <math>234 \times 100 = 23,400</math>      <math>0.234 \times 100 = 23.4</math>  <math>234 \times 1,000 = 234,000</math>      <math>0.234 \times 1,000 = 234</math> </p>	M	Hth	Tth	Th	H	T	O	Th	H	T	O	Tth	Hth	Thth					●●	●●	●●					●●	●●	●●
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
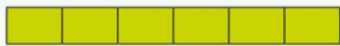
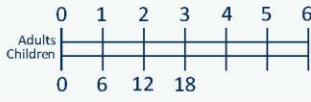
# Multiplication

<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Order of operations</b></p> <p>Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> 	
<p><b>Multiply decimals by integers</b></p> <p>This is the first time children multiply decimals by numbers other than 10, 100 or 1,000. Encourage them to make links with known facts and whole number multiplication.</p>	<p>I know that ... <math>\times</math> ... = ..., so I also know that ... <math>\times</math> ... = ...</p>  <p> <math>6 \times 2 = 12</math>      <math>6 \times 0.2 = 1.2</math> </p>	<p>I need to exchange 10 ... for 1 ...</p>  <p> <math>213 \times 4 = 852</math>      <math>2.13 \times 4 = 8.52</math> </p>

# Multiplication

Progression of skills	Key representations
<p><b>Multiply fractions by fractions</b></p> <p>Encourage children to give answers in their simplest form.</p>	<p>When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.</p>  <p> <math>\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}</math>                <math>\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}</math>                <math>\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}</math> </p>
<p><b>Find the whole</b></p> <p>Children multiply to find the whole from a given part.</p>	<p>If <math>\frac{1}{3}</math> is ... , then the whole is ... <math>\times</math> ...</p> <p><math>\frac{1}{3}</math> of <math>\underline{\quad} = 18</math></p>  <p><math>18 \times 3 = 54</math></p> <p><math>\frac{1}{3}</math> of <math>54 = 18</math></p> <p>If <math>\frac{4}{9}</math> is ... , then <math>\frac{1}{9}</math> is ... and the whole is ... <math>\times</math> ...</p> <p><math>\frac{4}{9}</math> of <math>\underline{\quad} = 48</math></p>  <p> <math>\frac{1}{9} = 48 \div 4 = 12</math>  <math>9 \times 12 = 108</math>  <math>\frac{4}{9}</math> of <math>108 = 48</math> </p>

# Multiplication

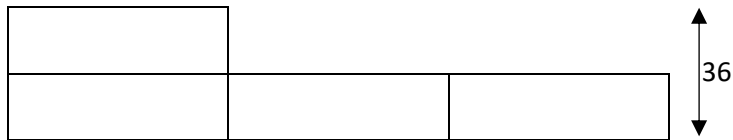
Progression of skills	Key representations																																
<p><b>Calculate percentages</b></p> <p>Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</p>	<p>There are ... lots of ... % in 100% To find ... % , I need to divide by ...</p> <table border="1" data-bbox="523 1243 826 1310"> <tr><th colspan="4">100%</th></tr> <tr><td>50%</td><td></td><td>50%</td><td></td></tr> <tr><td>25%</td><td>25%</td><td>25%</td><td>25%</td></tr> </table> <p>50% of ... = ... <math>\div</math> 2 25% of ... = ... <math>\div</math> 4</p> <p>... % is made up of ... % , and ... %</p> <table border="1" data-bbox="858 1232 1308 1288"> <tr><th colspan="10">100%</th></tr> <tr><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td></tr> </table> <p>To find 30%, I can find 10% and then multiply it by 3 To find 23%, I can use 10% <math>\times</math> 2 and 1% <math>\times</math> 3 To find 99%, I can find 1%, then subtract from 100%</p>	100%				50%		50%		25%	25%	25%	25%	100%										10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
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<p><b>Calculations involving ratio</b></p> <p>Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.</p>	<p>For every ... , there are ...</p> <p>For every 1 adult on a school trip, there are 6 children.</p> <p>adults </p> <p>children </p> <table border="1" data-bbox="1061 1467 1260 1590"> <tr><th>Adults</th><th>Children</th></tr> <tr><td>1</td><td>6</td></tr> <tr><td>2</td><td>12</td></tr> <tr><td>3</td><td>18</td></tr> </table> <p>The ratio of adults to children is 1 : 6</p> 	Adults	Children	1	6	2	12	3	18																								
Adults	Children																																
1	6																																
2	12																																
3	18																																

## Year 6 Division

### Pictorial

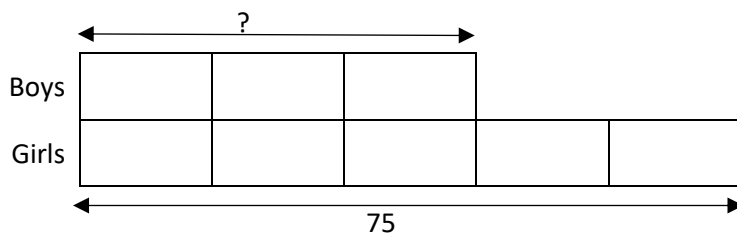
Bar modelling continues to be used to solve problems, including comparing problems:

The sum of two numbers is 36. The larger number is 3 times the smaller number. What are the two numbers?



(The four equal divisions must add up to 36 and therefore each box should contain 9. The two numbers are 9 and 27).

There are  $\frac{3}{5}$  as many boys as girls. If there are 75 girls, how many boys are there?



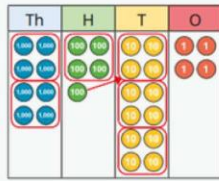
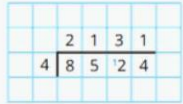
(The five equal divisions for the girls result in each box being worth 15. Therefore the 3 boxes for the boys add up to 45).

*There is a clear link between these bar modelling approaches and how ratio and proportion can be modelled effectively with the bar approach.*

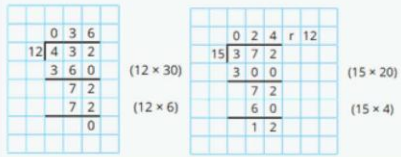
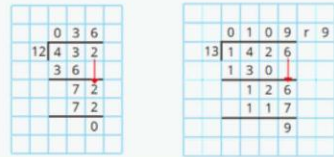







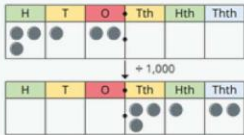

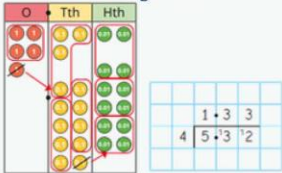
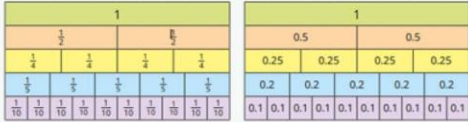
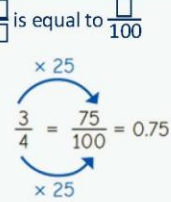
# Division

<b>Year 6</b>	<ul style="list-style-type: none"> <li>Perform mental calculations, including with mixed operations and large numbers.</li> <li>Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.</li> <li>Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.</li> <li>Divide numbers by 10, 100 and 1,000 giving answers up to three decimal places.</li> <li>Use written division methods in cases where the answer has up to two decimal places.</li> <li>Associate a fraction with division and calculate decimal fraction equivalents.</li> <li>Divide proper fractions by whole numbers [for example, <math>\frac{1}{3} \div 2 = \frac{1}{6}</math>]</li> <li>Solve problems involving the calculation of percentages.</li> </ul>
<b>Progression of skills</b>	<b>Key representations</b>
<b>Short division</b>  Encourage children to interpret remainders in context, for example knowing that "4 remainder 1" could mean 4 complete boxes with 1 left over so 5 boxes will be needed.	There are ... groups of ... hundreds/tens/ones/ in ... I can exchange 1 ... for 10 ...  




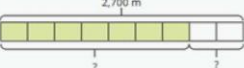
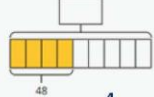
# Division

<b>Progression of skills</b>	<b>Key representations</b>	
<b>Mental strategies</b>  Include partitioning and number line strategies outlined in Y5 as well as division using factors.	To divide by ... , I can first divide by ... and then divide the answer by ...  $240 \div 60 = 240 \div 10 \div 6$ $240 \rightarrow +10 \rightarrow \square \rightarrow +6 \rightarrow \square$  $480 \div 24 = 480 \div 4 \div 6$ $480 \rightarrow +4 \rightarrow \square \rightarrow +6 \rightarrow \square$	
<b>Long division</b>  The long division method is introduced for the first time. Two alternative methods are shown.	<b>Method 1</b> 	<b>Method 2</b> 
<b>Order of operations</b> Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.	... has greater priority than ..., so the first part of the calculation I need to do is ...   	

# Division

Progression of skills	Key representations
<p><b>Divide by 10, 100 and 1,000</b></p> <p>Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.</p>	<p>To divide by ... , I move the digits ... places to the right.</p>  <p> <math>312 \div 10 = 31.2</math>  <math>312 \div 100 = 3.12</math>  <math>312 \div 1,000 = 0.312</math> </p> <p> <math>906 \div 10 = 90.6</math>  <math>906 \div 100 = 9.06</math>  <math>906 \div 1,000 = 0.906</math> </p>
<p><b>Divide decimals by integers</b></p> <p>This is the first time children divide decimals by numbers other than 10, 100 or 1,000</p>	<p>I know that ... <math>\div</math> ... = ..., so I also know that ... <math>\div</math> ... = ...</p>  <p> <math>39 \div 3 = 13</math>  <math>3.9 \div 3 = 1.3</math>  <math>0.39 \div 3 = 0.13</math> </p> <p>I need to exchange 1 ... for 10 ...</p> 
<p><b>Decimal and fraction equivalents</b></p>	<p>The fraction ... is equivalent to the decimal ...</p>  <p> <math>\frac{1}{5} = 0.2</math>   <math>\frac{2}{5} = 0.4</math>   <math>\frac{3}{5} = 0.6</math> </p> <p><math>\frac{3}{4}</math> is equal to <math>\frac{\square}{100}</math></p>  <p> <math>\frac{3}{4} = \frac{75}{100} = 0.75</math> </p>

# Division

Progression of skills	Key representations
<p><b>Divide a fraction by an integer</b></p> <p>This is the first time children divide fractions by an integer.</p>	<p>... ones divided by 2 is ... ones so ... sevenths divided by 2 is ... sevenths.</p>  <p> <math>\frac{4}{7} \div 4 = \frac{1}{7}</math>  <math>\frac{4}{7} \div 2 = \frac{2}{7}</math> </p> <p>I am dividing by ..., so I can split each part into ... equal parts.</p>  <p> <math>\frac{1}{3} \div 2 = \frac{1}{6}</math> </p> <p>... is equivalent to ... so ... <math>\div</math> ... = ... <math>\div</math> ...</p>  <p> <math>\frac{2}{3} = \frac{4}{6}</math>  so <math>\frac{2}{3} \div 4 = \frac{4}{6} \div 4 = \frac{1}{6}</math> </p>
<p><b>Fraction of an amount</b></p> <p>Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.</p>	<p>To find <math>\frac{1}{\square}</math> I divide by ...</p> <p> <math>\frac{1}{2}</math> of 36 = <math>36 \div 2</math>  <math>\frac{1}{12}</math> of 36 = <math>36 \div 12</math> </p> <p>If <math>\frac{1}{\square}</math> is equal to ..., then <math>\square</math> are equal to ...</p>  <p> <math>\frac{7}{9}</math> of 2,700 = <math>\frac{1}{9}</math> of 2,700 <math>\times 7</math> </p> <p>If <math>\frac{\square}{\square}</math> is equal to ..., then the whole is equal to ...</p>  <p> <math>\frac{4}{9}</math> of <math>\square = 48</math> </p>

# Division

Progression of skills	Key representations																																																											
<p><b>Calculate percentages</b></p> <p>Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</p>	<p>There are ... lots of ... % in 100%</p> <p>To find ... %, I need to divide by ...</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="4">100%</td></tr> <tr><td>50%</td><td>25%</td><td>25%</td><td>50%</td></tr> <tr><td>25%</td><td>25%</td><td>25%</td><td>25%</td></tr> </table> <p>50% of ... = ... ÷ 2 25% of ... = ... ÷ 4</p>	100%				50%	25%	25%	50%	25%	25%	25%	25%	<p>... % is made up of ... %, and ... %</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="10">100%</td></tr> <tr><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td><td>10%</td></tr> </table> <p>To find 30%, I can find 10% and then multiply it by 3 To find 23%, I can use 10% × 2 and 1% × 3 To find 99%, I can find 1%, then subtract from 100%</p>	100%										10%	10%	10%	10%	10%	10%	10%	10%	10%	10%																										
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<p><b>Calculations involving ratio</b></p> <p>Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.</p>	<p>For every ... , there are ...</p> <p>For every 6 children on a school trip, there is 1 adult.</p> <p>adults <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td> </td></tr></table></p> <p>children <table border="1" style="display: inline-table; vertical-align: middle;"><tr><td> </td><td> </td><td> </td><td> </td><td> </td><td> </td></tr></table></p> <div style="text-align: right; margin-top: 20px;"> <table border="1" style="border-collapse: collapse;"> <tr><td colspan="2" style="text-align: center;">+ 6</td></tr> <tr><td style="text-align: center;">Adults</td><td style="text-align: center;">Children</td></tr> <tr><td style="text-align: center;">1</td><td style="text-align: center;">6</td></tr> <tr><td style="text-align: center;">2</td><td style="text-align: center;">12</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">18</td></tr> <tr><td colspan="2" style="text-align: center;">+ 6</td></tr> </table> <p style="margin-top: 10px;"> <table style="border-collapse: collapse;"> <tr><td style="text-align: right;">0</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">1</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">2</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">3</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">4</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">5</td><td style="border-left: 1px solid black; width: 10px;"></td><td style="text-align: left;">6</td></tr> <tr><td style="text-align: right;">Adults</td><td></td><td style="text-align: right;">6</td><td></td><td style="text-align: right;">12</td><td></td><td style="text-align: right;">18</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td style="text-align: right;">Children</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table> </p> </div> <p>The ratio of children to adults is 6 : 1</p>									+ 6		Adults	Children	1	6	2	12	3	18	+ 6		0		1		2		3		4		5		6	Adults		6		12		18							Children												
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